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Original Research

Using literacy strategies to assist parents with Common Core mathematical practices

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Abstract: Parent and family engagement in Common Core State Standards (CCSS) emphasizes meaningful involvement and consultation of parents in supporting their children with necessary mathematics and literacy skills at home. As the mathematical methods taught in schools following the CCSS differ from what parents are used to, parents need support learning new approaches to solving mathematics problems – moving from an algorithm-concentrated method to a concept-oriented process – and helping their children develop necessary mathematics and literacy strategies. This study describes a parental intervention designed by researchers to support parents using literacy strategies to help their fourth-grade (nine- to ten-year-old) children with mathematics homework. The results from the parent interviews indicate that the parental intervention program enabled parents to be aware of their beliefs and attitudes about mathematics, school mathematics, ways to support children's mathematical work, and knowledge and practice of active involvement to connect literacy and mathematics. This paper discusses the implications for educators in leveraging language and literacy strategies for parents' involvement in their children's mathematical thinking and provides recommendations for further research.

Keywords: Literacy, Mathematics education, Homework, Parent program

Introduction

A group of mathematicians set out to build a mathematics curriculum that works for the future, which we now call Common Core State Standards (CCSS) (National Governors Association Center for Best Practices, 2010) for mathematics. The most widely credited researchers for the CCSS for mathematics are Coleman and Zimba (2008), appointed by the Chief of State School Officers (CCSSO) and the National Governors Association (NGA). By 2009, the CCSS for mathematics were widely published across the United States in various mathematics textbooks. The standards were written to ensure that all students in every state were learning the same curriculum at the same pace; therefore, if a fourth grader from New Jersey moved to Ohio, she would be on track with the curricula. In June 2010, the New Jersey State Board of Education adopted the CCSS in mathematics.

The CCSS for mathematics aim to underline students' developing number sense skills and justify the pertinent mathematical concepts behind formulas and traditional algorithms. Many of the CCSS for mathematical practices involve visualizing problems and their solutions, either by drawing them out or using hands-on items (i.e., manipulatives), which help students better understand

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what numbers symbolize and engage in reasoning and problem-solving processes in various logical ways. It took a while for teachers and students to get acquainted with this new way of learning mathematics (Pomerantz & Kempner, 2013).

The CCSS maintain that parental support is essential to student achievement in mathematics. Parental involvement also benefits elementary teachers and their teaching of mathematics by reinforcing what mathematics has been taught at home, increasing students' motivation for, their engagement with, and positive attitudes toward learning mathematics, overcoming possible mathematical learning gaps and individual challenges, supporting a positive classroom environment, and identifying mathematical difficulties and promptly creating a mathematical intervention plan (Budhrani et al., 2021; Chen et al., 2024; Tan et al., 2020). Considering these benefits, parents need to be involved in their children's mathematical learning to promote their mathematics achievement by indirectly assisting the mathematics teaching process in the classroom. Nevertheless, different parental involvement programs result in different gains in students' mathematics learning (Patall et al., 2008). The literature indicates a need for further research to closely examine the quality of various parental involvement programs and how parents are involved in different involvement activities (Hoover-Dempsey et al., 2005; Panaoura, 2021).

In New Jersey, United States, parents are expected to be involved in school and district-level education plans, which provide the foundation regarding how CCSS will shape the delivery of educational services provided for elementary and secondary school students. However, schools were not provided with substantive assistance regarding how to help parents understand the new concepts that the standards bring. Parents have a wide range of mathematical knowledge: Some have mathematics or teaching background and have been prepared for STEM or educational careers; others have no such background, making it difficult to assist their children with mathematics homework. Furthermore, many parents still struggle with the new way of teaching and learning mathematics (Leyva et al., 2019). Parents are more used to the traditional way of teaching and learning mathematics, where the focus is on achieving the correct answer through repetition and drills to ensure foundational mastery – but not on fully grasping the concepts behind arithmetic. In this study, we hypothesize that using literacy strategies could assist parents in understanding the CCSS while working on their children's mathematics homework.

Consistent with the integrated curriculum approach, there has been a movement to link mathematics with literacy to provide a more meaningful context for mathematics problem-solving (Unal et al., 2023). Speaking and listening are the most common literacy activities in mathematical intervention studies (Chapin & O'Connor, 2022). Therefore, it is vital for parents to regularly and strategically engage in conversations with children during literacy and mathematics activities. For instance, children should discuss the meaning of relevant mathematics vocabulary and have chances to practice using these words with support during their mathematics instruction at school (Carter et al., 2024).

Hendrix et al. (2019) synthesized that all informal mathematical intervention studies require child participants to read, underscoring the importance of teachers and parents intentionally supporting students' reading skills within mathematics activities (Ekawati et al., 2022). Additionally, in over 70% of the mathematical interventions that Carter et al. (2024) reviewed, an essential part of the program contained daily reading.

This review points to the necessity of educators and parents to purposefully support students with reading as they engage children in mathematics activities. Children need to regularly read and discuss mathematics problems and have others (i.e., teachers and parents) involved in their mathematics learning process (Leyva et al., 2019). Since all the informal mathematical intervention studies for children from preschool to third grade include some aspects of literacy, it is crucial to prioritize an integrated approach to learning mathematics with an emphasis on literacy skills also in the upper elementary grades (Ekawati et al., 2022); this means that teachers in upper grades could develop similar interventions for parents and students to connect mathematics and literacy in informal learning settings (Unal et al., 2023). To actualize this goal, workshops could be created and scheduled after school or early evenings for parents and students so that they could be engaged in mathematical and literacy activities together.

According to Peters et al. (2008), educators should create space and opportunities for parents and their children to work together; the more parents are invited into schools, the more their children succeed at home. Authors (2017) point out that parental participation in their children's learning at school results in statistically significant differences in students' academic scores. Parent-child shared book literacy programs (Authors, 2022) and family mathematics nights (Nantais & Skyhar, 2022) are examples of parents and children working together for students' educational gains. With the CCSS for mathematics introducing new vocabulary and problem-solving strategies that parents may not have been familiar with, it is difficult for parents to explain new mathematical concepts to their children using their mathematics background. Research also shows that parents are influential in their children's confidence in learning mathematics and their beliefs in the usefulness and importance of mathematics (Gün et al., 2023; Mohr-Schroeder et al., 2017). Parents notice mathematics' usefulness and necessity through parental involvement programs and help their children change their attitudes toward mathematics (Hodge & Lawson, 2018; Young & Reed, 2023). Accordingly, through interventions, educators and researchers should encourage parents to offer critical support to help their children develop positive mathematics dispositions.

There is a paucity of research regarding after-school

interventions in which parents and their children work together on mathematics homework, guided by a teacher or a researcher. We aimed to address this gap by conducting five after-school programs, where parent and child dyads were taught using the same mathematics curriculum the children used in school (which followed the CCSS for mathematics). The researchers added literacy strategies to assist the parents and children in deconstructing the language of mathematics, simplifying the mathematics task, and looking at what was being asked by the question. The following research question guided this multiple case study: How do parents' beliefs and ideas on using literacy strategies to assist their children's solution to mathematics word problems change with a five-week university-based school-based intervention?

Literature review

Mathematics at home

Parental involvement in communication purposes is beneficial for both elementary and middle school students' educational outcomes (Otani, 2022). Parental involvement for monitoring purposes is especially effective for elementary school students. Schools and educators reach out to parents and promote both forms of involvement according to the child's school level by, for example, handing out letters or booklets and organizing interventions. Policymakers also instruct teachers about parental involvement and encourage parents to get involved.

For many families, reading stories is a regular part of a child's home routine and parental involvement. Parents are mainly motivated to read stories to their children because they believe this activity promotes children's school interests (Jeynes, 2010). However, some parents pay much less attention to supporting their children's mathematical work than their involvement with their children's reading (Drummond & Stipek, 2005). Berkowitz (2018) states the following about parents' beliefs regarding their children's mathematics education:

A widely held belief among parents is that children's math education is primarily theresponsibility of schools, and that their role in supporting their children's math learning is not as important as their role in supporting their children's reading […] This belief is reinforced by messages conveyed through the media and schools, which predominantly focus on the need for parents to interact with their children around language and reading […] (pp. 75-76).

Unfortunately, the notion that mathematics education is the purview of the schools ignores that mathematical input at home is an important predictor of children's mathematical success (Lawson, 2003; Sheldon & Epstein, 2005).

According to the CCSS (2010) for mathematics, students are expected to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others" (p. 8). Students are additionally expected to "analyze and evaluate the mathematical thinking strategies of others" and to "use the language of mathematics to express mathematical ideas precisely" (National Governors Association Center for Best Practices, 2010, p. 10). Students need to use their linguistic and mathematical knowledge to practice these mathematical reasoning skills and succeed in mathematics (Otani, 2020).

The CCSS (2010) for mathematics include problem-solving, reasoning and proof, connections, communication, and representation. To meet these standards, students must develop the ability to think with language and literacy concepts and communication skills (Shanahan, 2012). As Vygotsky (1978) posited, "talking about a given concept does not merely reflect thought, but it generates new thoughts and new ways to think" (p. 35). We can deduct the same argument for mathematics: talking and reflecting on mathematical concepts and vocabulary could enhance one's mathematical reasoning and creativity in solving mathematics problems. In that respect, literacy offers methods for teachers and students to read and understand mathematics problems, write and draw their way of mathematical thinking, and communicate effectively (Koponen et al., 2020).

Parents play a vital role in helping their children develop a positive attitude toward mathematics by making it part of their daily interactions. Encouraging children to ask questions, solve problems, and explore mathematics in various contexts fosters a sense of curiosity and confidence in their mathematical abilities. Additionally, parents can model mathematical thinking and provide hands-on learning experiences that make mathematics more engaging and relevant to children's lives (Leyva et al., 2019).

Similarly, parents' use of mathematical language with their children, such as using number words in mealtime conversations or during play, positively predicts children's mathematics abilities (Gunderson & Levine, 2011). In addition to these direct opportunities for exposure to mathematics content, some evidence suggests that parents indirectly contribute to their children's mathematics learning through their beliefs or attitudes about mathematics. For example, some parents believe that concrete arithmetical skills are more valuable than oral discussions of mathematical problems (Hawighorst, 2005). Such parental beliefs create a discrepancy between what mathematics students experience at school and what mathematics is appreciated at home.

The difficulty some parents may experience in supporting their children's mathematics at home derives from the newer curriculum currently used for teaching mathematics in the classroom. Bay-Williams et al. (2016) suggest that many parents are baffled by the mathematics curriculum aligned with the CCSS, which is very different from the mathematics curriculum parents were taught in the past, which focused only on teaching algorithms to find the correct answer. The CCSS for mathematics provide a deeper understanding of

mathematics concepts than just memorizing formulas. The emphasis on conceptual understanding can confuse parents who learned mathematics with formulas and rote-driven methodologies (Civil, 2006; Muir, 2021; Peressini, 1998; Pritchard, 2004; Quintos et al., 2005).

Research in mathematics education shows that families typically use their funds of knowledge (i.e., practical skills and knowledge that families utilize in their daily lives) to teach mathematical skills and pedagogically different strategies from those used in the classroom (González et al., 2005; Jay et al., 2017). As they are involved with their children's mathematics learning, families usually participate in basic problem-solving activities, different from the mathematical concepts typically taught in today's mathematics classrooms (Sun & Moreno, 2021). However, there is evidence suggesting that literacy and language skills are the baseline for parents to become familiar with new ways of learning mathematics (i.e., using literacy strategies before traditional algorithms) and help them with CCSS and the mindset behind those mathematics standards (Jay & Xolocotzin, 2013; Leyva et al., 2019). Addressing multiple content standards within the school day is complex, especially if we consider mathematics, language, and literacy discrete subjects. However, researchers (Jay et al., 2017; Levya et al., 2019 & Shanahan, 2012) know that each learning domain is essential and builds on each other over time. Students cannot be successful at learning mathematics without also being able to read, write, and speak. Secondly, an essential component of CCSS for mathematics involves word problems, which require reading comprehension to understand the context and details necessary to solve these problems. Parents with good language and literacy skills can help their children decompose the meaning behind these problems, identify the information in the word problems necessary to solve them and develop solution strategies (Ginsburg et al., 2012).

Disciplinary literacy tasks are highly positioned within CCSS to encourage the development of content knowledge, reading, writing, and higher-order thinking skills across grades and within each content area (Zygouris-Coe, 2021). For example, in each content area, such as mathematics, teachers are expected to use literacy strategies such as close reading (Baldwin & Morrow, 2019), a focus of the CCSS across grades and content areas. For example, when using close reading in mathematics, teachers direct students to read each portion of the mathematics problem to determine what keywords are in the mathematics problem and cite specific evidence from the problem when writing or speaking to support their conclusions (Oh et al., 2022).

Mathematics and literacy

It is established that students' mathematics outcomes are associated with their reading abilities (Ünal et al., 2023). However, researchers do not understand why reading and mathematical executive functioning skills co-develop at similar rates (Geary et al., 2017). Children's early reading skills predict their later competencies across various mathematical domains (e.g., arithmetic, algebra, geometry) (Duncan & Magnuson, 2011). The relationship between mathematics and reading may be due to combined reliance on similar skills, such as efficiently matching meaning-based units (e.g., language sounds and quantities) with corresponding random symbols (Rousselle & Noël, 2007). For instance, shortfalls in literacy skills (e.g., slow word-reading development, low phonological awareness) also predict poor mathematics development (Jordan et al., 2010; Melby-Lervåg et al., 2012). Likewise, children's early mathematics skills predict their later reading outcomes (Claessens et al., 2009; Duncan & Magnuson, 2011).

The literature unequivocally establishes that the quality of the home environment and the opportunities parents provide at home are crucial for a child's learning (Kwing-Cheung & McBride, 2017). As the primary influencers in a child's early learning, parents have a significant role in creating a home learning environment that fosters the child's awareness of daily mathematics experiences. Engaging in mathematics activities at home tailored to the child's developmental stage is a powerful way for parents to contribute to their child's mathematics knowledge (Zippert & Rittle-Johnson, 2020).

Children's mathematics skills are positively affected by the activities they perform with their parents, such as laying games with blocks, cooking or engaging in measurement activities, and reading storybooks emphasizing the development of children's mathematical vocabulary (Turker-Biber et al., 2021). Parents support their children's mathematics skills in daily life with formal and informal numerical activities. Formal activities (e.g., working with numerical activity books and explaining complex mathematics concepts while students work on their mathematics homework [Kwing-Cheung & McBride, 2017]) and informal activities (e.g., playing card games, singing songs, and measuring the ingredients while cooking [Vandermaas-Peeler et al., 2009]) are used by parents for supporting children's mathematical skills. The shared storybook reading as an informal activity allows the parent and child to discuss different mathematical ideas. The shared storybook reading is described as reading books with children, not reading books to the children (Han & Neuharth-Pritchett, 2014). Shared storybook reading is commonly studied for its literacy benefits, including opportunities to decode print, understand story structures, and learn new vocabulary terms (Hendrix et al., 2019). Exposure to written language can improve children's language abilities and give them the essential skills to comprehend written texts. However, storybook reading also has the potential to provide a meaningful context for the learning of mathematics (van den Heuvel-Panhuizen et al., 2014). Reading with children combines two critical tools, text, and illustrations, which can show visual depictions of mathematics ideas, present problems for

children to explore, and provide a model for mathematics investigations.children to explore, and provide a model for mathematics investigations.

Parental beliefs and attitudes

Parental beliefs and attitudes influence the child's development and the parent-child relationship (Sigel et al., 2014). Parental attitudes and beliefs about schools and their children's learning can vary; however, the one constant is that parents believe that learning occurs best in school, which can influence their interactions with their children (Chen et al., 2024). The impact of culture and parents' own experiences shape parental attitudes and beliefs about education, affecting how much parents participate in school-related activities and how much they assist with their children's homework.

Parents are role models for their children, and their attitudes toward learning can directly communicate and influence their children's learning interests during parent-child interactions (Eccles et al., 1998; Wigfield et al., 2015). For example, when children see their parents reading at home, they tell their children that they value reading. Parents' attitudes toward their own learning include their feelings when learning mathematics and literacy, including enjoyment and anxiety for either subject (Del Río et al., 2017; Susperreguy et al., 2020). Parents' attitudes toward literacy may indicate enjoying reading with their children or reading a book or a magazine (Susperreguy et al., 2020). Alternatively, parents' attitudes about mathematics can also model apprehension about completing mathematics homework with their children (Del Río et al., 2017).

Oh et al. (2022) suggested another developmental pathway: negative emotions experienced by mathematics-anxious parents while assisting their children with mathematics homework could result in parents engaging in controlling behaviors to ensure their children finish their work quickly. The quicker children finish, the less time the parents would have to endure the negative feelings associated with mathematics (Kwing-Cheung & McBride, 2017).

In their study, Cheung et al. (2023) studied parents' beliefs about the nature of mathematics, categorizing them as either static (i.e., mathematics is a subject consisting of "terms, rules and procedures") or dynamic (i.e., mathematics is a "process of human inquiry") (p. 14). The researchers found that parents who view mathematics as a dynamic subject are more inclined to employ indirect methods (i.e., providing hints, asking questions) to teach their children numeracy skills (Cheung et al., 2023). Building on these findings, researchers recommend that future research should examine parents' beliefs about the nature of mathematics or literacy, how those beliefs affect students' mathematical development and design and test parental interventions that take parental beliefs into account (Cheung et al., 2023; Sonnenschein et al., 2012).

Methods

Procedures

The researchers in this study designed a program to support parents in understanding how to use new mathematics skills and their children's mathematical backgrounds. The study underlined the use of literacy strategies to assist their children in solving mathematics problems in the fourth-grade curricula. One of the recurring themes was to encourage parents to think about the literacy strategies already taught to their children in earlier grades and build upon that knowledge.

Through our intervention, we aimed to determine how the absence of parental involvement in the CCSS for mathematics might be addressed by implementing a five-session literacy and mathematics intervention, which was designed to increase parents' knowledge of the new ways to do mathematics (found within the CCSS for mathematics) with their children and assist parents by having them apply known literacy strategies (see Table 1), such as recalling, visualizing, rereading, when working with their students in mathematics. The researchers aimed to tap into parents' prior knowledge of literacy terms, which could be related to solving mathematics problems in connection with their knowledge of everyday mathematics experiences. The five-week after-school intervention was collaboratively designed between one mathematics faculty and two literacy faculty members, providing guidance for four parents and their children in implementing literacy strategies to effectively make sense of and solve mathematics word problems with attention to parental involvement procedures.

The series of workshops were offered to all fourth-grade parents in an elementary school in a Northeastern part of the United States. In collaboration with the school's principal, we sent recruitment emails to all parents of fourth-grade students. Only four parents were interested in participating in the research and attended all five intervention sessions. In other words, we used convenience sampling to recruit participants for the study. Each parent had a bachelor's degree. One parent received her entire education in Eastern Europe, another parent was a former teacher, one parent was not working, and the last held a master's degree in business administration and a master's in accounting.

The parents and their children who participated in the intervention attended five weekly sessions, each lasting 60 minutes. The researchers followed the mathematics curriculum that the children used in school. The researchers reviewed the school's mathematics curriculum and the CCSS for mathematics assigned to the children during the intervention. For example, since fourth-grade students in the school started to learn about invented strategies, we intentionally shaped the mathematical focus of the program on these topics.

After reviewing the mathematics curriculum, the university researchers selected weekly motivational literacy topics that reflected what the children were being

taught in their classroom and what corresponded with the mathematics topics. During the weekly training sessions with the parents, the university researchers incorporated the following instructional procedures: 1) modeling the language strategies that correlated with the mathematical assignments their children received at school, 2) demonstrating how the mathematics homework could be supported by first approaching each problem by looking at known language and literacy concepts, and 3) including mathematics concepts from everyday life. All three of these approaches were incorporated into each 60-minute session. In each session, the parents and children worked on one mathematics assignment selected from their child's mathematics homework assignment.

During each session, the four (N = 4) parents and each of their children were provided with instructions on how to (1) use literacy strategies to solve mathematics problems and (2) use their everyday mathematics knowledge to apply to their school mathematics. (see Table 1 for the description of the weekly session topics). The researchers also distributed A Family's Guide, published by the National Council of Teachers of Mathematics (Mirra, 2004), to help the parents incorporate literacy strategies and everyday mathematics into their time after the researchers finished their lectures. To facilitate the parents' understanding and appreciation of mathematics, the researchers encouraged parents to be positive about mathematics and working with mathematics, link mathematics with daily life by providing concrete everyday examples, make mathematics fun with engaging discussions and games, talk about mathematics-related careers (e.g., economists, statisticians, engineers, mathematics teachers), have high expectations in all cases, and support homework but do not complete it for them.

Table 1. Description of weekly session topics

Sessions	Mathematics focus	Literacy Strategies	Parental Strategies to use at Home during Homework
1st Session	Mental mathematics to add and subtract big numbers	Connections between mathematics and literacy in early grades	Underlying the importance of parental involvement
2nd Session	Breaking apart numbers for multiplication	Building and using vocabulary	Linking mathematics to daily life and helping with homework
3rd Session	Multiplying with area model (one digit × two digits)	Determining important segments in word problems	Using questions to promote student thinking
4th Session	Multiplying with area model (two digits × two digits)	Visualizing ideas	Justifying the solution
5th Session	Distributive property of multiplication	Making inferences and judgments	Sharing different strategies and reflecting on new confidence to help with homework

During our sessions, we expect parents to help their children with their homework by asking the following questions to their children. Following the recommendations by Mirra (2004), these questions were intentionally crafted for parents to help students' problem-solving abilities using literacy strategies:

What is the problem you are working on?
What do the directions say?
What words or phrases are you familiar with that can help you solve the problem?
Can you explain what the teacher asked you to do?
Where do you think you should begin?
Where can we find help in your textbook or notes?
Can you draw a picture or make a diagram?

The study supported parents in seeking alternatives to school-centered definitions of mathematical vocabulary terms, starting with supporting parents to find mathematics in their everyday lives, such as in cooking, at the grocery store, and paying bills. Once parents felt more knowledgeable about new mathematical strategies, our primary goal was to encourage parents to use their literacy strengths to help them figure out what troubled them about school mathematics so that they could feel more confident assisting their children with their mathematics homework. The sessions were held between pairs of each parent and child before coming together as a whole group. This allowed for greater levels of participation and helped parents develop confidence in expressing their thoughts with other parents before sharing them with a larger group. During the first and second sessions, parents drew or wrote down some of their thoughts on paper and explored different mathematical topics and strategies.

A significant component of the programs was the meetings held after the parents spent time with their children. After the parents worked with their children on mathematical activities, the researchers scaffolded the mathematical content and the parents' experiences with their children. This potent combination of supporting parents in using the literacy strategies they knew and the mathematical experiences they were already familiar with (e.g., balancing a checkbook and keeping scores in sports) helped illuminate the rationale for participating in the program.

The researchers conducted pre- and post-interviews with parents before and after the program. The data gathered from

the interviews is crucial for ensuring the research findings' quality, depth, and reliability (Paratore, 2021; Patton, 2014). Participants might recall additional information or develop new insights upon reflecting on their responses from the pre-interview to the post-interview, allowing them to reflect on their initial responses, potentially uncovering more nuanced insights.

In this paper, we report findings regarding the changes in parents' involvement in their children's mathematical learning based on the data collected from pre- and post-interviews with each parent. During the interviews, we asked parents about their experiences and attitudes toward mathematics as younger students and adults. The researchers also asked the parents how involved they were in helping their children with mathematics homework. Pre- and post-interviews also included a mathematical problem for parents to work on and explain how they would help their children solve the problem and their attitudes and beliefs about working with their children with their mathematics homework (see Appendix for interview protocol). Table 2 provides the mathematics problems asked of parents during each interview:

Table 2. Mathematical tasks used during the interview

	The problems	Instructions
Pre-interview task	Jamie has 126 marbles. Her friend gave her 58 more. Jamie tries to figure out how many marbles she has now. She adds 58 to 126 and says 1714. Do you think Jamie is correct? Explain .	What kind of knowledge would your child need to successfully complete this problem?
Post-interview task	Mark has 15 cartons of juice bottles. Each tartons holds 24 bottles of juice. To find the total number of the juice bottles, Mark thinks he needs to multiply 24 with 15. He breaks 24 and 15 into tens and ones: 24×15=(20+4)×(10+5). 20×10=200. 4×5=20. He say there will be 200+20=220 juice bottles in 15 cartons. Do you think Mark is correct? Explain.	How would you help child make sense of the problem?

Data analysis

Our research utilized a multiple case study methodology where each parent behaved as a unique case throughout our parental involvement intervention (Yin, 2009). A thematic analysis approach was used to organize our qualitative data into manageable codes, categories, and themes (Peel, 2020). More specifically, holistic coding of each case data was used to understand the bounded phenomenon in depth (i.e., the effect of our five-week intervention on parents' support for children's use of language and literacy skills to deconstruct mathematics problems and develop more confidence in helping their children) (Corbin & Strauss, 1990). Using these approaches, we examined interview transcripts of each parent's case to identify themes and make sense of and interpret parents' reasoning about the ways to support their children's mathematical problem-solving with literacy strategies across different contexts.

Data from pre- and post-interviews and their transcriptions were coded through a series of iterations bound by the research question. The data were coded using the values and process coding strategies (Saldaña, 2009). The researchers first conducted open-coding procedures to identify critical variables of the parents' improvement (i.e., process, practices) from pre- to post-interview in using literacy and everyday mathematics experiences to assist their children with their mathematics (Corbin & Strauss, 1990). We also explored the parents' thoughts (i.e., beliefs, values, attitudes, and knowledge) about mathematics, school mathematics, and teaching mathematics.

We conducted a series of data analysis meetings, during which we narrowed down the codes and categories and collapsed data into themes. The initial list of codes, categories, and themes was compared and aligned with the theory and literature on how using literacy phrases, and common mathematical knowledge can guide parents in assisting their children (Baker & Street, 2004). The list of codes, categories, and themes were individually tested, from which the results of the first comparison of initial codes assisted in determining the final code list.

We identified all the relationships of all codes and categories and conceptualized the findings as we related them to the themes that emerged from the data. One priori theme deliberately focused on identifying each parent's level of active involvement with literacy strategies to support their children's mathematical work. We defined active involvement for this study as parents (1) asking questions, (2) reading the entire mathematics assignment, and (3) dialoguing with their child about it before attempting to solve the mathematics problem. All data were then analyzed to determine the number of meaning segments (i.e., a portion of parent interview responses that convey a distinct idea) for each code, category, and theme. The first two authors analyzed the interviews separately and achieved a 92% interrater reliability agreement. Data from the transcribed interviews were finally analyzed to draw

out vignettes that best represented the themes regarding the parents' change in their participation level and attitudes to assist and scaffold their children's mathematics homework (Anfara et al., 2002).

Findings

Our qualitative data analysis revealed 232 meaning segments from the pre- and post-interviews of the four parents who participated in our five-week parental involvement program. In Table 3, we provide the distribution of meaning segments for each code, category, and theme in 2 layers: the number of parents and meaning segments.

Table 3. Distribution of qualitative themes, categories, and codesz

Themes	Categories	Codes	Pre-interview	Post-interview
Parental beliefs about mathematics, learning, and their involvement	Beliefs and attitudes about mathematics	Past struggle in mathematics	2 (3)	1 (1)
		Post-interview	2 (2)	0 (0)
		Confidence in mathematics	1 (1)	2 (2)
		Having mathematics anxiety	2 (3)	0 (0)
		Embracing a product-oriented view of mathematics	2 (8)	2 (3)
		Embracing a process-oriented view of mathematics	0 (0)	2 (2)
	Beliefs and attitudes about school mathematics	Negative attitudes about Common Core mathematical practices	3 (3)	0 (0)
		Appreciating new ways of doing mathematics	2 (5)	2 (8)
		Comparing old and new mathematics	4 (4)	2 (4)
	Beliefs about ways to support children's mathematical work	Capability based on grade level and expectations	2 (2)	1 (2)
		Sharing the old strategy as an option	4 (4)	0 (0)
		Teaching the old strategy for efficiency	2 (5)	0 (0)
		Assisting only when asked	3 (3)	0 (0)
		Checking for accuracy	3 (5)	2 (4)
	Beliefs about factors impacting children's learning	Viewing the homework as children's responsibility	3 (6)	1 (3)
		The impact of the pandemic	2 (7)	0 (0)
		Children's selective interest in mathematics	2 (3)	0 (0)
		Children's selective interest in reading/writing	2 (2)	0 (0)
		Viewing children as independent learners	2 (4)	0 (0)
		Blaming others	2 (4)	0 (0)

	Knowledge about connections between literacy and mathematics	Conceptualizing literacy to visualize abstract mathematical concepts in real life	0 (0)	2 (12)
		Conceptualizing literacy as a means for language comprehension and vocabulary use in mathematical problem-solving	0 (0)	4 (15)
Parental knowledge and practice for children's mathematical development with literacy strategies	Practice of literacy strategies for mathematics	Guidance to underline important segments in problem statements	1 (1)	2 (3)
		Guidance to visualize abstract mathematical concepts in problem statements	1 (2)	2 (4)
		Guidance to analyze the context, evaluate the reasoning, and develop mathematical inferences	0 (0)	3 (5)
	Practice of active Involvement	Asking facilitatory questions	2 (9)	4 (5)
		Asking mathematics-specific operational questions	1 (1)	1 (2)
		Guidance to read/re-read problem statements	1 (1)	3 (7)
		Providing partial answers to problems	3 (3)	0 (0)
		Deferring to the old strategy	3 (3)	0 (0)
		Guidance to explain the mathematical vocabulary in problem statements	0 (0)	2 (2)
		Validly analyzing the mathematical work in problem statements	0 (0)	3 (3)
Other gains for parents and children from the process	Program benifits	Parents implementing Common Core mathematical practices	N.A.	4 (15)
		Parents' appreciating Common Core mathematical practices	N.A.	4 (21)
		Parents being more cooperative and involved in children's mathematics learning	N.A.	3 (7)
		Changes in children's attitudes towards mathematics	N.A.	2 (2)
		Changes in children's repertoire of mathematical strategies	N.A.	2 (2)
		Changes in children's social skills to learn mathematics	N.A.	2 (4)

Note. The first number represents the number of parents (N1 = 4) for that specific code; the second number in parentheses represents the number of meaning segments (N2 = 232) for that specific code. 94 and 138 meaning segments were identified from pre- and post-interviews, respectively.

After examining our data for themes and codes, we found three themes: (1) parental beliefs about mathematics, learning, and their involvement, (2) parental knowledge and practice for children's mathematical development with literacy strategies, and (3) other gains for parents and children from the process of their involvement in our intervention. In the following three subsections, we share these three major themes through parent interviews that we derived from the distribution of codes.

Parental beliefs about mathematics and their involvement in homework

While transcripts of the interviews indicated more emphasis on a product-oriented view of mathematics, this trend changed to a process-oriented view of mathematics in the post-interview. Considering the beliefs and attitudes about school mathematics, whereas the pre-interviews showed evidence of parents' negative attitudes about Common Core mathematical practices, they started to become more positive about these practices in the post-interviews by appreciating new ways of doing mathematics in today's mathematics classrooms.

For example, during the pre-interview, Parent 2 shared her view of mathematics with an emphasis on finding correct answers for the given mathematics problems and using memorization for success (i.e., embracing a product-oriented view of mathematics):

Mathematics [...] is one of the most important subjects. I think [mathematics] is more likely to develop your brain. That makes you build connections to the logical stuff and memorize things. Mathematics is a very foundational discipline. [...] You could memorize stuff and then build upon it instead of thinking about it using logic, so you save time in finding the correct answer. [Parent 2, Pre-interview]

Parent 2 also shared her negative beliefs and attitudes about school mathematics, specifically how students learn mathematics compared to how she learned basic operations from a traditional algorithmic stance. While the parent did not explicitly refer to the CCSS for mathematics in her statement, we coded her statement as her negative beliefs about the school mathematics and expectations with the implementation of CCSS in schools:

Interviewer: *Are there times when you teach your child methods for solving problems? Are these methods different from what they learn in school?*

Parent 2: *Yes, division is a different format and more complex than it should be. [For] division, I would teach her my way as far as I remember the standard algorithm. I would say [she] should memorize the multiplication table instead of trying to use logic.*

Interviewer: *Do you mean [equal size] grouping for multiplication or division?*

Parent 2: *I do not even think [equal size] grouping is critical to learning. Memorization and repetition are more important.*

During the post-interview, the same parent started to think about mathematics differently and slightly changed her view: "Mathematics is not just about memorization. It is more about conceptualizing and solving problems. To solve the [mathematics] problems, you need to know what it is about and think of different strategies [Parent 2, Post-Interview]".

Parent 2 also indicated in the post-interview that our program helped them make sense of the CCSS for mathematical practices and that students need to engage in these practices for mathematical understanding (i.e., appreciating new ways of doing mathematics). When we asked what she thinks about how mathematics is taught these days, Parent 2 replied as below:

Early on [during the program], we spent quite some time understanding new standards. I guess I better understand how to do it … Less [steps are] required for the kids. That is the thing that I gained, for sure. [...] It is different, but they are grasping it. I think she will see she is doing better than what I would offer her, so […] The program offers some good ideas about what they learn at school and how they have been taught, and now I can help with homework.

During the pre-interviews, parents indicated they either teach or prefer to share a traditional mathematical strategy they learned in their school years as an option while assisting their children's mathematical work. Neither of these codes emerged within the parents' post-interview transcripts.

There was one way we learned, and they had different ways of learning. I try to find what best suits them, but I understand where they come from. I am unsure if it is the right way, but it gives them an option. [Parent 1, Pre-interview, code: sharing the old strategy as an option]

I teach [my child] the way I learned. It was easier for me to explain it and quicker to find the answer. I found myself teaching them the way I learned it. [Parent 3, Pre-Interview, code: teaching the old strategy for efficiency]

Our analyses of the pre-interviews also revealed parents' beliefs regarding the factors they think influence their children's learning:

I feel like it is [learning] is their responsibility. It does not mean they do it, but it should be their responsibility. [Parent 1, Pre-interview, code: viewing the homework as children's responsibility]

I noticed that American schools do not encourage more work. The requirements are quite loose. [...] However, again, it [her child's learning] depends on the teacher very much. I saw different teachers, and she was doing different work qualities, and it depends on the requirements. If the teacher is stricter and requires more, she will do more, but if she is not, she does not. [Parent 2, Pre-interview, code: blaming others]

Virtual learning impacted her, as she lost interest in any work, especially remote work. [Parent 3, Pre-interview, code: the impact of the pandemic]

We interpret these three parents' statements above and all the codes in this category (i.e., beliefs about factors) as their reliance on externalization and abdication of responsibility for their children's (lack of or limited) learning rather than accepting their involvement. Pre-interview data showed more evidence of this whole category than post-interview data.

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Parental knowledge and practice

Each week, the researchers presented parents with different literacy strategies they could utilize while helping their children with mathematics tasks; parents' post-interview data revealed knowledge development and their ability to practice with this knowledge. With the completion of our program, parents started to conceptualize literacy as a discipline that could help students visualize abstract mathematical concepts in real life and as a means for language comprehension and vocabulary use in mathematical problem-solving. Neither of these codes emerged within the parents' pre-interview transcripts when we asked parents to share what they know regarding the connection between literacy and mathematics.

To assess parents' practices and the use of literacy strategies during their active involvement in homework with their children, we requested parents to work on a mathematics problem (see Figure 1) and explain how they would help their children solve the problem. During the post-interviews, parents used practices of literacy strategies discussed at the workshops, such as determining important segments in word problems, visualizing ideas, and developing mathematical inferences and judgments about the context. The frequency of these three codes was lower in the pre-interview data. We share excerpts from post-interviews for each of the codes below:

She also has to understand and visualize the vocabulary part. 15 times 24 is one thing, but you must understand that it is 15 cartons of juice bottles. You also have to draw it down. [Parent 1, post-interview, code: guidance to visualize]

It is helpful for them to have vocabulary to understand the context of story problems. So, I first ask Mary [her child] to understand and ensure she understands the context of what is being asked in this problem and vocabulary. Knowing the vocabulary in this problem equals what is being asked for regarding the operation. [Parent 4, post-interview, codes: guidance to analyze the context, guidance to explain the mathematical vocabulary]

The parent participants reported that at the end of the five-week intervention, they actively used language and literacy strategies to discuss the mathematics assignments at home before attempting to complete them. The parents' active involvement in some codes had increased at the post-interview compared to the pre-interview. For example, more parents asked facilitatory questions to help their children work with the given problem during the post-interview. Additionally, post-interviews revealed that more parents asked their children to reread the problem, help them explain the mathematical vocabulary, and validly analyze the mathematical work given. The frequency of these codes was lower in the pre-interview data:

A visual of that or how I would do it…other multiplication. Where you know you would take 15, 24. 5 fours. I would ask her: "What would you use?" Alternatively, "What method do you feel the most comfortable with?". And then she would start. She would either go that way [standard algorithm] or draw that. I feel this [area model] is what she feels most comfortable with. [...] So, you know you have got to do 20 times 10 and 4 times 5. But Mark is missing 4 times 10 and 20 times 5. So, she is going to add a couple more. [Parent 1, Post interview, codes: asking facilitatory questions, validly analyzing the mathematical work]

First of all, Mark is not correct. You must do 20 by 10, plus 20 by 5, and 4 by 10, and 4 by 5. Each number has to be multiplied. [Parent 2, post-interview, code: validly analyzing the mathematical work]

[In the given problem], I will circle it and say, "Just read it again - you did not pay attention to what it asked." [Parent 3, post-interview, code: guidance to re-read the problem]

I would first ask: Can you slow down? Can you read that again? Let us read this out loud. Can I hear what you are doing? [Parent 4, post-interview, codes: asking facilitatory questions, guidance to re-read the problem]

In these excerpts, parents utilize the new mathematical strategies and representations they learned during the intervention and guide their children in this respect instead of prioritizing what they are used to or comfortable with (i.e., standard algorithm). In a way, these excerpts show the benefit of our approach in the intervention: Parents who do not have strong confidence in mathematics can learn mathematics with their children, which in turn could enable students to reinforce their mathematical knowledge as they behave as mathematics teachers for their parents (Ginsburg et al., 2008). Additionally, parents apply literacy strategies when they (hypothetically) plan and strategize to help their children with mathematics word problems. Whereas the first two parents' excerpts include making sense of the mathematical task given during the interview, the last two parents' questions focus on literacy strategies without discussing mathematics concepts. Parents may need more support in making their questions more complex about mathematical concepts (Uscianowski et al., 2020).

Other gains for parents and children from our intervention

During the post-interviews, parents also expressed that, after the five interventions, they felt empowered to complete mathematics homework with their children. The following is an exemplary quote from one parent during the post-interview:

It was motivating and empowering for me to feel like I could apply my strengths in literacy to help my daughter

when she is struggling. Because these area-model and related mathematics problems are difficult to solve. But it did not seem undoable when we took a minute to breathe, read, and understand it. I felt like I was motivating her for the first time in my life with me (and myself). [Parent 3, post-interview, code: parents being more cooperative and involved, changes in children's social skills to learn mathematics]

All parents were motivated by being in a group, three of whom indicated that they became more cooperative with other parents and involved in children's mathematics learning. Parent 3, during the post-interview, stated: "I liked the idea of working with parents from the school; even if I did not know them personally, it was good to see we were all in the same boat." Parent 4 also stated, "When we started, Mary [her child] would not let me in. She wanted to do it on her own. But later, this changed. This program allowed us to work together. It integrated me into her homework" [post-interview, code: parents being more cooperative and involved].

Data from parents' discussions shows that many of the members began the workshops with the idea that they had little or no way to support their child at home with their mathematics homework or learning due to a lack of knowledge about new standards for mathematics and having a negative disposition towards mathematics in general or the standards specifically. Parents knew that mathematics was a subject with right or wrong answers; the consequence of this belief was that they never spoke about mathematics other than to inquire whether the children's homework was done. Post-interview data negated this belief and showed evidence that parents could implement and appreciate the CCSS for mathematical practices more. When we asked about their willingness to help their child with their mathematics homework as a result of their participation in our program, Parent 4 replied as below:

Absolutely. I understand now. Common Core has been a black cloud over our heads for so long. Thus, now, I see, understand, and think it makes sense. It is a revelation for me. [...] We could apply what we learned here to what we did with that. [...] I am an accountant, and I am just really thrilled that I did this [area model for multiplication] today. I have a better understanding [of the area model].

This excerpt indicates that Parent 4's involvement in our program helped them change their mathematical behaviors. With its increased emphasis on communication, critical thinking, problem-solving, and analytical thinking over rote memorization, the CCSS for mathematics may require many parents to interact with mathematics in ways different from their own mathematics education experiences (*Jackson & Remillard, 2005; Jay et al., 2017*).

Discussion

Parents demonstrated progress in the program by learning to prompt their children and ask them questions using familiar literacy terms and phrases to assist them in solving mathematics problems. Findings from the interviews show that parents preferred to direct their children to read the given mathematics tasks before rushing to solve the problem. In the post-interviews, when we asked them how they would help their children with the mathematics homework, they explained that they would coach their children to look carefully through the problem and point out any familiar words or phrases that might clarify it. During our five-week intervention, the parent-child partners frequently dialogued about their mathematical reasoning, communicating their thoughts to each other. We believe these interactions became critical in helping parents reshape their beliefs and reconsider their problem-solving approaches. It is possible that, as the parent-child pairs became more flexible in their thinking (i.e., leaving the 'old ways' of solving mathematics problems behind) from the first to the last intervention session, parents demonstrated more confidence in their mathematical reasoning and use of CCSS for mathematics practices during the post-interviews.

Our study reveals a noteworthy finding: parental involvement in programs or interventions that equip them with the requisite skills and strategies to comprehend the CCSS for mathematics, along with applying requisite literacy skills, increases their tendency to engage in their children's mathematics homework. Additionally, the more positive attitudes parents exhibit with their children, the better learning results students will gain in literacy and mathematics (Chen et al., 2024). Our findings are similar to those of Mangram and Metz (2018). In their study, Mangram and Metz provided a five-session intervention to parent-child dyads to help their children engage in Common Core Mathematical Practices. After the intervention, parents and children engaged more in Common Core Mathematical Practices. Parental interventions similar to ours also help parents reorient their beliefs about mathematics. For instance, Holtzman and her colleagues (2023) designed a program for parents, sharing fun mathematics activities with parents for 12 weeks via text messages. Parents' exposure to such mathematics activities enabled them to hold positive beliefs about mathematics and increased their assistance in their children's mathematics homework. In our study, participating parents similarly reshaped their beliefs about mathematics with the completion of our program.

There are some limitations to this program. First, only four parents participated. Second, the program only lasted five weeks, and finally, it was conducted in an upper-middle-class population. Future research studies might improve these limitations; the researchers might conduct this parent program with a larger group of parents in a culturally and linguistically diverse population for a longer time. Additionally, future research could tie these parental programs implemented by elementary preservice teachers. That way, while parents' attitudes about teaching and learning mathematics and curriculum could be enhanced, teacher candidates develop tacit knowledge regarding

implementing interdisciplinary mathematics and literacy lessons and experience concrete parental involvement strategies.

Conclusion

This study examined how an intervention design emphasizing literacy strategies can help parents rethink their beliefs about mathematics, literacy, the CCSS for mathematics, and parental practices when assisting their children with mathematics at home. We believe findings from our research could expand the knowledge of working more closely with parents to help them change their beliefs in assisting their children with mathematics homework using literacy strategies. Parental involvement in supporting children's mathematical reasoning is not a subject that has been widely researched; our data provide some evidence that parents, once engaged in a supportive intervention, could develop relevant beliefs to be actively involved in helping and enhancing their children's mathematical thinking. In designing such parental involvement programs, university researchers should collaborate with classroom teachers to identify the correct literacy strategies correlating with specialized mathematical vocabulary. Both parents and classroom teachers could then help their students develop the necessary metacognitive skills to guide them in understanding the mathematical text (Unal et al., 2023).

For our program's purpose, we underlined the use of mathematical practices from CCSS with literacy strategies to show parents how to assist their children's mathematics learning and provide support with mathematics at home. Nevertheless, since the adoption of the CCSS for mathematics in the U.S., parents experienced difficulties negotiating school-centered definitions and approaches to mathematics (Jay & Xolocotzin, 2013). There is little discussion in the literature regarding why parental involvement interventions may fail. According to Carter et al. (2022), one reason may be that too few programs involve a comprehensive design approach. In light of our findings, in designing parental involvement programs, researchers should make explicit efforts to help parents understand students' learning objectives, have hands-on experience with those objectives from a learner's perspective, and build trust and respect for different curricular initiatives (Hendrix et al., 2019).

Author's contributions

Kerry Carley Rizzuto contributed to the conceptualization of the paper, project administration, original data preparation, writing, reviewing, editing, and creating the literature about literacy education. Lilly M. Steiner contributed to conceptualizing the paper, writing, reviewing, editing, project administration, and designing literacy resources. Vecihi Serbay Zambak contributed to the project administration, methodology, design of the mathematical resources, data analysis, and writing, reviewing, editing.

Conflict of interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

Appendix

The appendix of this research is available at https://file.luminescience.cn/RPPE-277%20Appendix.pdf.

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