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Original Research

Fuzzy approach to analyzing a three-factor experiment with binary levels

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Abstract: A factorial experiment is a widely used statistical analysis technique in experimental studies in agricultural and engineering sciences. The classical factorial experiment design considers several factors at different crisp levels, which are quantitatively measurable. However, factor levels are often imprecise, vague, incomplete, or described linguistically. In such cases, fuzzy statistics are used instead of classical statistics when analyzing data with uncertain observations. In this paper, we propose an extension of the classical factorial design for analyzing experiments with fuzzy observations. The experimental data are represented using triangular fuzzy numbers and transformed using the α-cut interval approach, resulting in lower-level and upper-level factorial models. These models are analyzed separately to evaluate the effects of factors and their interactions under uncertainty. The results are also compared with classical ANOVA applied to defuzzified data to highlight the advantages of incorporating uncertainty through fuzzy representations. Furthermore, the robustness of the proposed approach is examined through sensitivity analysis across different α levels. The findings demonstrate that the proposed fuzzy factorial approach provides a more flexible framework for analyzing experimental data in the presence of imprecision and uncertainty.

Keywords: Design of experiments, Fuzzy environment, Factorial experiment, Fuzzy logic, Lower-level model, Upper-level model

1. Introduction

In any experiment where more than one factor is used and all levels of each factor are combined with all levels of every other factor, the experiment is called a factorial experiment. The concept of factorial experiments represents a major advancement in the design of experimental systems. In these designs, the effects of individual factors are calculated using the entire set of observations. This method can improve the efficiency of the experiment. Fisher [1] was the first to introduce the concept of factorial experiments, suggesting that factorial design is more efficient than studying one factor at a time. Yates [2] made significant contributions to the analysis of factorial designs through the Yates' method. Williams [3] provided an overview and interpretations of the interaction terms present in factorial experiments. Addelman [4] discussed developments in the design of factorial experiments before the 1960s. Factorial design has been applied and expanded in various fields (e.g., agriculture, medicine, engineering, industry, biology) by many authors. Patterson [5] specified

Received: Jan. 1, 2026; Revised: Mar. 22, 2026; Accepted: May 18, 2026; Published: Jun. 4, 2026

DOI: https://doi.org/10.55976/dma.42026151443-59

procedures for producing asymmetrical factorial designs based on the relationships between the amounts of treatment and plot components. Liao [6] proposed three classes of optimal designs with equal or unequal block sizes in two-level factorial experiments. Additional literature related to factorial experiments can be found in references [7, 8].

A crucial limitation is that the factorial experiment is used only when measured data are available in exact form. Observations from any experiment are often uncertain and contain some vagueness caused by human judgment, evaluations and decisions, measurement devices, environmental conditions, or the subjective or linguistic nature of the observed data. Therefore, the existing factorial experiment under classical statistics cannot be used when the observations are uncertain. Methods in fuzzy statistics provide a useful methodology to address these types of uncertainty and imprecision in the data. Numerous approaches have been proposed in the last decade to combine the classical design of experiments with fuzzy statistics. Zadeh [9] was the first to introduce fuzzy set theory. Romer and Kandel [10] investigated the effects of fuzzy data on statistical tests for hypothesis testing of fuzzy hypotheses. Buckley and Eslami [11] presented a method that estimates the variance and mean of fuzzy statistics. This method utilizes a set of confidence intervals to produce an estimator as a triangular number. Dubois and Prade [12] discussed algebraic operations by applying the fuzzification rule to fuzzy numbers. Arnold [13] defined fuzzy hypothesis testing for crisp data and proposed one- and two-sided hypothesis testing under a fuzzy environment for type I and type II errors. The fuzzy test for hypothesis testing under vague circumstances was proposed by Grzegorzewski [14]. The author also developed a fuzzy decision method that shows a scale of acceptability for binary hypotheses. Grzegorzewski and Mrowka [15] formulated operators to approximate fuzzy numbers to trapezoidal expected intervals. Buckley [16] compared the fuzzy means of m populations, where each population was normally distributed with the same but unknown variance. Wu [17] discussed hypothesis testing for fuzzy analysis of variance (ANOVA) using fuzzy data, introducing the concepts of optimistic and pessimistic degrees for h-level sets and solving the associated optimization problem. Lubiano and Trutsching [18] introduced the R package "Statistical Analysis of Fuzzy Data (SAFD)" for fuzzy ANOVA, using the de-fuzzification process for Fuzzy Random Variables. Jorge [19] described fuzzy least squares regression and combined it with two-way ANOVA to present a claim reserving method. Parchami et al. [20] extended the concept of one-way ANOVA for imprecise observations. Rahul et al. [21] proposed a post-hoc test for fuzzy one-way analysis of variance to identify the exact pairs of fuzzy means that are significant. Thakur et al. [22] proposed one-way analysis of covariance under a fuzzy environment and its application in drug yield analysis. Recent developments in fuzzy statistical analysis and uncertainty modeling further highlight the importance of fuzzy-based approaches in handling imprecise data (Garg [23]; Kahraman et al. [24]). Recent studies have also emphasized the role of fuzzy methods in decision-making and experimental analysis under uncertainty (Więckowski et al. [25]; Tang & Ahmad [26]).

In many real-world experimental situations, observed responses are not always recorded as precise numerical values. Measurement instruments, environmental variability, and subjective evaluations may introduce uncertainty or vagueness into experimental observations. Classical factorial experiment models assume exact observations and therefore may not adequately represent such uncertainty. To address this limitation, fuzzy set theory provides an effective framework for modeling imprecise and uncertain data. By integrating fuzzy numbers with factorial experimental design, it becomes possible to analyze treatment effects while explicitly accounting for uncertainty in observations. The primary reason for conducting this study is to address issues in studies utilizing imprecise and uncertain data that necessitate the application of the fuzzy 2^3 factorial experiment. For example, to determine the effect of different types of fertilizers on potato crop yield, consider three fertilizers: Nitrogen (N), Potash (K), and Phosphate (P), each having two fuzzy levels, high and low. The quantities at each level are inexact, which creates vagueness in the data. Therefore, a fuzzy 2^3 factorial experiment is necessary for this type of experiment. Although several researchers have developed statistical methods for fuzzy data, most existing studies focus on fuzzy hypothesis testing, fuzzy regression, and fuzzy one-way analysis of variance. Very little attention has been given to factorial experimental designs under fuzzy environments, particularly for multi-factor experiments involving interactions between factors.

To the best of our knowledge, a systematic framework for analyzing three-factor factorial experiments with fuzzy observations has not been adequately addressed in the literature. This study aims to fill that gap by proposing a fuzzy factorial design methodology. The data are represented as triangular fuzzy numbers. The fuzzy factorial design is defuzzified using the α-cut interval approach, resulting in lower and upper-level models. Both models are discussed separately and fuzzy logic is used to draw conclusions. We implement and explain this method with a numerical example. Compared to existing methods in classical statistics, the proposed fuzzy factorial method is adaptable and efficient in the presence of uncertainty.

1.1 Objectives of the study

The main objectives of this study are:

1) To extend the classical 2^3 factorial experiment framework to experimental observations represented as fuzzy numbers.

2) To develop a methodology based on the α-cut approach for transforming fuzzy observations into lower and upper factorial models.

3) To analyze main and interaction effects under fuzzy environments using factorial ANOVA.

4) To evaluate the robustness of the proposed approach through sensitivity analysis across different α levels.

This paper is structured as follows. Section 2 discusses some basic definitions of fuzzy numbers and concepts related to the research. After defuzzification, the separate analysis of the lower and upper fuzzy 2^3 factorial experiment is presented in Section 3. Section 4 provides and explains the illustration. A comparison with the existing model is made in Section 5 and discussed in Section 6. The final section presents the conclusion of the paper.

2. Preliminaries

Definition 2.1 Generalized fuzzy number [22]

A fuzzy subset $\widetilde{A} = (p, q, r, s{:}\ \omega)$ defined on the real line $\Re$ is said to be a generalized fuzzy number if its membership function $(\mu_{\widetilde{A}}(x))$ satisfies the following properties:

a) $\mu_{\widetilde{A}}(x) = 0$ for all $x \in (-\infty, p]$.

b) $\mu_L(x) = l_{\widetilde{A}}(x)$ is strictly increasing on $[p, q]$.

c) $\mu_{\widetilde{A}}(x) = \omega$ for all $x \in [q, r]$.

d) $\mu_R(x) = r_{\widetilde{A}}(x)$ is strictly decreasing on $[r, s]$.

e) $\mu_{\widetilde{A}}(x) = 0$ for all $x \in [s, \infty)$.

f) $\mu_{\widetilde{A}}(x)$ is a continuous mapping from $\Re$ to the closed interval $[0, \omega]$.

where $0 \leq \omega \leq 1$; p, q, r, and s are real numbers with $p < q \leq r < s$. $\widetilde{A}$ is a fuzzy number with $\mu_{\widetilde{A}}(x) \in \Re$. $l_{\widetilde{A}}(x)$ and $r_{\widetilde{A}}(x)$ are the left and right functions of the fuzzy numbers, respectively.

Definition 2.2 Triangular fuzzy numbers (TrFNs) [22]

A TrFN is a set of three real numbers (p, q, r) such that $p < q < r$, where p, q, and r are the lower, central, and upper values of the TrFN, respectively. The membership function of a triangular fuzzy number can be expressed as:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0\,; & x < p \\ \dfrac{x-p}{q-p}; & p \leq x \leq q \\ \dfrac{r-x}{r-q}; & q \leq x \leq r \\ 0\,; & x > r \end{cases}$$

Definition 2.3 α-cut [22]

A widely used tool for handling fuzzy numbers is the α-cut. For any fuzzy number $\widetilde{\mathcal{A}}$, the α-cut is defined as

$$\mathcal{A}_\alpha = \{x \in X;\ \mu_{\widetilde{\mathcal{A}}}(x) \geq \alpha\}$$

Where $\mathcal{A}_\alpha$ is a non-fuzzy set and $\alpha \in [0, 1]$.

It can be seen from the definition of a fuzzy number that an α-cut interval of fuzzy number is a closed interval. So for a fuzzy number $\widetilde{\mathcal{A}}$, we have

$$\mathcal{A}_\alpha = [\mathcal{A}_l^\alpha, \mathcal{A}_u^\alpha]$$

where

$$\mathcal{A}_l^\alpha = \inf\{x \in X; \mu_{\widetilde{\mathcal{A}}}(X) \geq \alpha\}, \text{ for all } \alpha \in [0,1]$$

i.e., the left endpoint of the fuzzy number $\widetilde{\mathcal{A}}$, and

$$\mathcal{A}_u^\alpha = \sup\{x \in X; \mu_{\widetilde{\mathcal{A}}}(X) \geq \alpha\}, \quad \text{for all } \alpha \in [0,1]$$

i.e., the right endpoint of the fuzzy number $\widetilde{\mathcal{A}}$.

Remark 2.1: Although triangular fuzzy numbers are used in this study for illustration because of their simplicity and ease of computation, the proposed fuzzy factorial framework is not limited to this specific representation. The methodology is fundamentally based on the α-cut representation of fuzzy numbers, which applies to a wide range of fuzzy data, including trapezoidal fuzzy numbers, interval-valued fuzzy numbers, and general LR-type fuzzy numbers. By replacing the corresponding α-cut bounds, the proposed approach can be directly extended to other fuzzy data representations without any structural modifications to the factorial design.

3. Fuzzy 2^3 factorial experiment

Let us consider three factors, A, B, and C, each with two fuzzy levels, resulting in $2^3 = 8$ treatment combinations. Let the corresponding fuzzy levels $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ denote the second level of each respective factor. The first fuzzy level of each factor A, B, and C is denoted by the absence of the corresponding letters $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$. The eight treatment combinations can be expressed as follows:

$\widetilde{a_o b_o c_o}$ or $\tilde{1}$: factor A, factor B, and factor C, all at the first fuzzy level.

$\widetilde{a_1 b_o c_o}$ or $\tilde{a}$: factor A at the second fuzzy level, and factor B and factor C at the first fuzzy level.

$\widetilde{a_o b_1 c_o}$ or $\tilde{b}$: factor B at the second fuzzy level, and factor A and factor C at the first fuzzy level.

$\widetilde{a_o b_o c_1}$ or $\tilde{c}$: factor C at the second fuzzy level, and factor A and factor B at the first fuzzy level.

$\widetilde{a_1 b_1 c_o}$ or $\widetilde{ab}$: factor A and factor B at the second fuzzy level, and factor C at the first fuzzy level.

$\widetilde{a_0 b_1 c_1}$ or $\widetilde{bc}$: factor B and factor C at the second fuzzy level, and factor A at the first fuzzy level.

$\widetilde{a_1 b_o c_1}$ or $\widetilde{ac}$: factor A and factor C at the second fuzzy level, and factor B at the first fuzzy level.

$\widetilde{a_1 b_1 c_1}$ or $\widetilde{abc}$: factor A, factor B, and factor C, all at the second fuzzy level.

The fuzzy 2^3 factorial design can be executed as a Fuzzy Completely Randomized Design with 8 treatments, a Fuzzy

Randomized Block Design with r replications, or a Fuzzy Latin Square Design. In a fuzzy 2^3 experiment, we split the fuzzy treatment sum of squares with 7 degrees of freedom into 7 orthogonal components corresponding to the three main effects ($\widetilde{A}$, $\widetilde{B}$, and $\widetilde{C}$), three first-order interactions ($\widetilde{AB}$, $\widetilde{AC}$, and $\widetilde{BC}$), and one second-order interaction $\widetilde{ABC}$, each carrying 1 degree of freedom.

3.1 Yates' method for computing fuzzy factorial effects

Yates provided a method to compute various effect totals for any 2^n experiments.

Step 1: List all possible combinations of fuzzy treatments in the first column.

Step 2: Record the fuzzy treatment totals from all replications in the second column, aligned with the appropriate fuzzy treatment combinations.

Step 3: Break the values of the second column into pairs starting from $\tilde{1}$. In the third column, write the sums of the pairs in the first half and the differences in the second half.

Step 4: After breaking down the third column's values into pairs, repeat step 3 to obtain the fourth and fifth columns.

Table 1 shows the total factorial effect calculations for 2^3 experiments. The computation of this rule accounts for the interactions between the treatment combinations and the main effects.

3.2 Sign table

The effects and interactions between composite and individual treatments in 2^3 factorial experiments are shown in Table 2, which also contains the divisor, where $\widetilde{M}$ is the general fuzzy mean.

To find any effect, simply multiply the sign in the table's column by the associated treatment combination, add the

Table 1. Yates' method for a fuzzy 2^3 experiment

Treatment combination (1)	Treatment total (2)	Treatment contrasts (3)	Effect contrasts (4)	Factorial effect estimates (5)	Effect total
$\tilde{1}$	$[\tilde{1}]$	$[\tilde{1}]+[\tilde{a}]=\tilde{\tau}_1$	$\tilde{\tau}_1+\tilde{\tau}_2=\tilde{Q}_1$	$\tilde{Q}_1+\tilde{Q}_2=\tilde{\kappa}_1$	Grand Sum
$\tilde{a}$	$[\tilde{a}]$	$[\tilde{b}]+[\widetilde{ab}]=\tilde{\tau}_2$	$\tilde{\tau}_3+\tilde{\tau}_4=\tilde{Q}_2$	$\tilde{Q}_3+\tilde{Q}_4=\tilde{\kappa}_2$	$[\widetilde{A}]$
$\tilde{b}$	$[\tilde{b}]$	$[\tilde{c}]+[\widetilde{ac}]=\tilde{\tau}_3$	$\tilde{\tau}_5+\tilde{\tau}_6=\tilde{Q}_3$	$\tilde{Q}_5+\tilde{Q}_6=\tilde{\kappa}_3$	$[\widetilde{B}]$
$\widetilde{ab}$	$[\widetilde{ab}]$	$[\widetilde{bc}]+[\widetilde{abc}]=\tilde{\tau}_4$	$\tilde{\tau}_7+\tilde{\tau}_8=\tilde{Q}_4$	$\tilde{Q}_7+\tilde{Q}_8=\tilde{\kappa}_4$	$[\widetilde{AB}]$
$\tilde{c}$	$[\tilde{c}]$	$[\tilde{a}]-[\tilde{1}]=\tilde{\tau}_5$	$\tilde{\tau}_2-\tilde{\tau}_1=\tilde{Q}_5$	$\tilde{Q}_2-\tilde{Q}_1=\tilde{\kappa}_5$	$[\widetilde{C}]$
$\widetilde{ac}$	$[\widetilde{ac}]$	$[\widetilde{ab}]-[\tilde{b}]=\tilde{\tau}_6$	$\tilde{\tau}_4-\tilde{\tau}_3=\tilde{Q}_6$	$\tilde{Q}_4-\tilde{Q}_3=\tilde{\kappa}_5$	$[\widetilde{AC}]$
$\widetilde{bc}$	$[\widetilde{bc}]$	$[\widetilde{ac}]-[\tilde{c}]=\tilde{\tau}_7$	$\tilde{\tau}_6-\tilde{\tau}_5=\tilde{Q}_7$	$\tilde{Q}_6-\tilde{Q}_5=\tilde{\kappa}_7$	$[\widetilde{BC}]$
$\widetilde{abc}$	$[\widetilde{abc}]$	$[\widetilde{abc}]-[\widetilde{bc}]=\tilde{\tau}_8$	$\tilde{\tau}_8-\tilde{\tau}_7=\tilde{Q}_8$	$\tilde{Q}_8-\tilde{Q}_7=\tilde{\kappa}_8$	$[\widetilde{ABC}]$

Table 2. Signs and divisors for each fuzzy factorial effect

Factorial effect	Treatment means								Divisor
	$(\tilde{1})$	$(\tilde{a})$	$(\tilde{b})$	$(\widetilde{ab})$	$(\tilde{c})$	$(\widetilde{ac})$	$(\widetilde{bc})$	$(\widetilde{abc})$	
$\widetilde{M}$	+	+	+	+	+	+	+	+	8
$\widetilde{A}$	-	+	-	+	-	+	-	+	4
$\widetilde{B}$	-	-	+	+	-	-	+	+	4
$\widetilde{AB}$	-	-	-	-	+	+	+	+	4
$\widetilde{C}$	+	-	-	+	+	-	-	+	4
$\widetilde{AC}$	+	-	+	-	-	+	-	+	4
$\widetilde{BC}$	+	+	-	-	-	-	+	+	4
$\widetilde{ABC}$	-	+	+	-	+	-	-	+	4

results, and divide the total by the corresponding divisor. For the main effect, use a plus sign when the corresponding factor is at the second level and a minus sign when it is at the first level. The algebraic sign for a second-order interaction can be obtained by multiplying the signs of $\widetilde{A}$, $\widetilde{B}$, and $\widetilde{C}$, or $\widetilde{BC}$ and $\widetilde{A}$, or $\widetilde{AC}$ and $\widetilde{B}$.

3.3 Statistical analysis of 2^3 factorial design under fuzzy data

The fuzzy 2^3 factorial experiment, in which the observations are in the form of triangular fuzzy numbers, is discussed here. By the α-cut method of defuzzification, we transform the fuzzy 2^3 factorial experiment model into two crisp models: the lower-level 2^3 factorial and the upper-level 2^3 factorial experiment models. Using the upper limit of the fuzzy interval obtained after defuzzification, we construct the upper-level crisp 2^3 factorial model, and using the lower limit of the fuzzy interval, we construct the lower-level crisp 2^3 factorial model. Finally, we analyze the lower- and upper-level models using the classical 2^3 factorial experiment technique.

Let us consider $\widetilde{x_{ijkl}}$ as the response observed corresponding to the ith level of A, the jth level of B, the kth level of C, and the lth replicate. The observations of treatment combinations are measured in terms of triangular fuzzy numbers, i.e., $(f_{ijkl}, g_{ijkl}, h_{ijkl})$, where $f_{ijkl} < g_{ijkl} < h_{ijkl}$. By using the α-cut defuzzification method, the lower-level limit is

$$\widetilde{x^l_{ijkl_\alpha}} = \left(g_{ijkl} + \left(g_{ijkl} - f_{ijkl}\right)\alpha\right)$$

and the upper-level limit is

$$\widetilde{x^u_{ijkl_\alpha}} = \left(h_{ijkl} - \left(h_{ijkl} - g_{ijkl}\right)\alpha\right)$$

The α-cut defuzzification approach plays a central role in transforming fuzzy observations into interval-valued data. For a given $\alpha \in [0, 1]$, each fuzzy observation is converted into a closed interval, resulting in two crisp factorial models: a lower-level (pessimistic) model and an upper-level (optimistic) model. The lower-level model represents conservative estimates of treatment effects, while the upper-level model represents optimistic estimates. This dual analysis enhances interpretability by providing bounds on factorial effects rather than a single point estimate, thereby allowing decision-makers to assess the robustness of conclusions under uncertainty. Finally, we formulate the ANOVA table for the lower-level and upper-level models separately.

3.3.1 Main effects and interactions in fuzzy 2^3 factorial design for the lower-level model

Let $[\tilde{1}^l_\alpha]$, $[\tilde{a}^l_\alpha]$, $[\tilde{b}^l_\alpha]$, $[\widetilde{ab}^l_\alpha]$, $[\tilde{c}^l_\alpha]$, $[\widetilde{ac}^l_\alpha]$, $[\widetilde{bc}^l_\alpha]$, and $[\widetilde{abc}^l_\alpha]$ denote the total lower yield of r units (plots) receiving the treatments $\tilde{1}$, $\tilde{a}$, $\tilde{b}$, $\widetilde{ab}$, $\tilde{c}$, $\widetilde{ac}$, $\widetilde{bc}$, and $\widetilde{abc}$, respectively. Let the corresponding lower-level mean values, obtained by dividing the totals by r, be denoted by $(\tilde{1}^l_\alpha)$, $(\tilde{a}^l_\alpha)$, $(\tilde{b}^l_\alpha)$, $(\widetilde{ab}^l_\alpha)$, $(\tilde{c}^l_\alpha)$, $(\widetilde{ac}^l_\alpha)$, $(\widetilde{bc}^l_\alpha)$, and $(\widetilde{abc}^l_\alpha)$, respectively. The simple lower effect of A is the difference in the lower mean yields of A due to increasing factor A from its lower fuzzy level $\widetilde{a^l_{o_\alpha}}$ its higher fuzzy level $\widetilde{a^l_{1_\alpha}}$, while factors B and C are held at fixed fuzzy levels. The simple lower effect of A is given in Table 3.

The lower main effect of A is defined as the average of the obtained simple lower effects. Thus,

$$A^l_\alpha = \frac{1}{4}\left[\left(\widetilde{abc}^l_\alpha\right) - \left(\widetilde{bc}^l_\alpha\right) + \left(\widetilde{ac}^l_\alpha\right) - \left(\tilde{c}^l_\alpha\right) + \left(\widetilde{ab}^l_\alpha\right) - \left(\tilde{b}^l_\alpha\right) + \left(\tilde{a}^l_\alpha\right) - \left(\tilde{1}^l_\alpha\right)\right]$$

$$A^l_\alpha = \frac{1}{4}\left[\left(\widetilde{abc}^l_\alpha\right) + \left(\widetilde{ac}^l_\alpha\right) + \left(\widetilde{ab}^l_\alpha\right) + \left(\tilde{a}^l_\alpha\right) - \left\{\left(\widetilde{bc}^l_\alpha\right) + \left(\tilde{c}^l_\alpha\right) + \left(\tilde{b}^l_\alpha\right) + \left(\tilde{1}^l_\alpha\right)\right\}\right]$$

Similarly, the lower main effects of factors B and C are obtained as

Table 3. Calculation for simple lower effect of A for the lower-level model

Lower fuzzy level of B	Lower fuzzy level of C	Simple lower effect of A
$\widetilde{b^l_{o_\alpha}}$	$\widetilde{c^l_{o_\alpha}}$	$\widetilde{a^l_{1_\alpha}}\widetilde{b^l_{o_\alpha}}\widetilde{c^l_{o_\alpha}} - \widetilde{a^l_{o_\alpha}}\widetilde{b^l_{o_\alpha}}\widetilde{c^l_{o_\alpha}} = (\tilde{a}^l_\alpha) - (\tilde{1}^l_\alpha)$
$\widetilde{b^l_{1_\alpha}}$	$\widetilde{c^l_{o_\alpha}}$	$\widetilde{a^l_{1_\alpha}}\widetilde{b^l_{1_\alpha}}\widetilde{c^l_{o_\alpha}} - \widetilde{a^l_{o_\alpha}}\widetilde{b^l_{1_\alpha}}\widetilde{c^l_{o_\alpha}} = (\widetilde{ab}^l_\alpha) - (\tilde{b}^l_\alpha)$
$\widetilde{b^l_{o_\alpha}}$	$\widetilde{c^l_{1_\alpha}}$	$\widetilde{a^l_{1_\alpha}}\widetilde{b^l_{o_\alpha}}\widetilde{c^l_{1_\alpha}} - \widetilde{a^l_{o_\alpha}}\widetilde{b^l_{o_\alpha}}\widetilde{c^l_{1_\alpha}} = (\widetilde{ac}^l_\alpha) - (\tilde{c}^l_\alpha)$
$\widetilde{b^l_{1_\alpha}}$	$\widetilde{c^l_{1_\alpha}}$	$\widetilde{a^l_{1_\alpha}}\widetilde{b^l_{1_\alpha}}\widetilde{c^l_{1_\alpha}} - \widetilde{a^l_{o_\alpha}}\widetilde{b^l_{1_\alpha}}\widetilde{c^l_{1_\alpha}} = (\widetilde{abc}^l_\alpha) - (\widetilde{bc}^l_\alpha)$

$$B_\alpha^l = \frac{1}{4}[\{(\widetilde{abc}_\alpha^l) + (\widetilde{ab}_\alpha^l) + (\widetilde{bc}_\alpha^l) + (\tilde{b}_\alpha^l)\}$$

$$- \{(\widetilde{ac}_\alpha^l) + (\tilde{c}_\alpha^l) + (\tilde{a}_\alpha^l) + (\tilde{1}_\alpha^l)\}]$$

$$C_\alpha^l = \frac{1}{4}[\{(\widetilde{abc}_\alpha^l) + (\widetilde{ac}_\alpha^l) + (\widetilde{bc}_\alpha^l) + (\tilde{c}_\alpha^l)\}$$

$$- \{(\widetilde{ab}_\alpha^l) + (\tilde{b}_\alpha^l) + (\tilde{a}_\alpha^l) + (\tilde{1}_\alpha^l)\}]$$

The lower average effect of A_α^l (with C fixed at one level) at the level $\widetilde{b_{o_\alpha}^l}$ of B is

$$\frac{1}{2}[(\widetilde{ac}_\alpha^l) - (\tilde{c}_\alpha^l) + (\tilde{a}_\alpha^l) - (\tilde{1}_\alpha^l)],$$

and at the level of $\widetilde{b_{1_\alpha}^l}$ of B is

$$\frac{1}{2}[(\widetilde{abc}_\alpha^l) - (\widetilde{bc}_\alpha^l) + (\widetilde{ab}_\alpha^l) - (\tilde{b}_\alpha^l)]$$

If factors A and B are independent, we can expect that the average effect of A will remain the same at either level of B. However, if factors A and B are not independent, the measure of their interaction AB is given by half the difference between the average effect of A at the second and first fuzzy levels of B. Mathematically,

$$AB_\alpha^l = \frac{1}{4}[\{(\widetilde{abc}_\alpha^l) - (\widetilde{bc}_\alpha^l) + (\widetilde{ab}_\alpha^l) - (\tilde{b}_\alpha^l)\}$$

$$- \{(\widetilde{ac}_\alpha^l) - (\tilde{c}_\alpha^l) + (\tilde{a}_\alpha^l) - (\tilde{1}_\alpha^l)\}]$$

$$AB_\alpha^l = \frac{1}{4}[\{(\widetilde{abc}_\alpha^l) + (\widetilde{ab}_\alpha^l) + (\tilde{c}_\alpha^l) + (\tilde{1}_\alpha^l)\}$$

$$- \{(\widetilde{bc}_\alpha^l) + (\tilde{b}_\alpha^l) + (\widetilde{ac}_\alpha^l) + (\tilde{a}_\alpha^l)\}]$$

Similarly, the expression for the interactions BC and AC as given below

$$BC_\alpha^l = \frac{1}{4}[\{(\widetilde{abc}_\alpha^l) + (\widetilde{bc}_\alpha^l) + (\tilde{a}_\alpha^l) + (\tilde{1}_\alpha^l)\}$$

$$- \{(\widetilde{ab}_\alpha^l) + (\tilde{b}_\alpha^l) + (\widetilde{ac}_\alpha^l) + (\tilde{c}_\alpha^l)\}]$$

$$AC_\alpha^l = \frac{1}{4}[\{(\widetilde{abc}_\alpha^l) + (\widetilde{ac}_\alpha^l) + (\tilde{b}_\alpha^l) + (\tilde{1}_\alpha^l)\}$$

$$- \{(\widetilde{ab}_\alpha^l) + (\tilde{a}_\alpha^l) + (\widetilde{bc}_\alpha^l) + (\tilde{c}_\alpha^l)\}]$$

From Table 3, the expressions for the interaction of AB_α^l at levels $\widetilde{c_{o_\alpha}^l}$ and $\widetilde{c_{1_\alpha}^l}$ of factor C are as follows:

Interaction of AB_α^l at level $\widetilde{c_{o_\alpha}^l}$ of factor C = $\frac{1}{2}[(\widetilde{ab}_\alpha^l) - (\tilde{b}_\alpha^l) - (\tilde{a}_\alpha^l) + (\tilde{1}_\alpha^l)]$.

Interaction of AB_α^l at level $\widetilde{c_{1_\alpha}^l}$ of factor C = $\frac{1}{2}[(\widetilde{abc}_\alpha^l) - (\widetilde{bc}_\alpha^l) - (\widetilde{ac}_\alpha^l) + (\tilde{c}_\alpha^l)]$.

Hence, the interaction effect of AB_α^l with C is half the difference of the above expressions:

$$ABC_\alpha^l = \frac{1}{4}[\{(\widetilde{abc}_\alpha^l) - (\widetilde{bc}_\alpha^l) - (\widetilde{ac}_\alpha^l) + (\tilde{c}_\alpha^l)\}$$

$$- (\widetilde{ab}_\alpha^l) + (\tilde{b}_\alpha^l) + (\tilde{a}_\alpha^l) - (\tilde{1}_\alpha^l)]$$

Table 2 can be used to express the various factorial effect totals for the lower level as mutually orthogonal contrasts of the eight treatment totals. Another convenient method is the Yates' method, which calculates the total factorial effects as:

$$[\tilde{A}_\alpha^l] = [\widetilde{abc}_\alpha^l] - [\widetilde{bc}_\alpha^l] + [\widetilde{ac}_\alpha^l] - [\tilde{c}_\alpha^l]$$

$$+ [\widetilde{ab}_\alpha^l] - [\tilde{b}_\alpha^l] + [\tilde{a}_\alpha^l] - [\tilde{1}_\alpha^l]$$

$$[\tilde{B}_\alpha^l] = [\widetilde{abc}_\alpha^l] + [\widetilde{bc}_\alpha^l] - [\widetilde{ac}_\alpha^l] - [\tilde{c}_\alpha^l]$$

$$+ [\widetilde{ab}_\alpha^l] + [\tilde{b}_\alpha^l] - [\tilde{a}_\alpha^l] - [\tilde{1}_\alpha^l]$$

$$[\widetilde{AB}_\alpha^l] = [\widetilde{abc}_\alpha^l] + [\widetilde{bc}_\alpha^l] + [\widetilde{ac}_\alpha^l] + [\tilde{c}_\alpha^l]$$

$$- [\widetilde{ab}_\alpha^l] - [\tilde{b}_\alpha^l] - [\tilde{a}_\alpha^l] - [\tilde{1}_\alpha^l]$$

$$[\tilde{C}_\alpha^l] = [\widetilde{abc}_\alpha^l] - [\widetilde{bc}_\alpha^l] - [\widetilde{ac}_\alpha^l] + [\tilde{c}_\alpha^l]$$

$$+ [\widetilde{ab}_\alpha^l] - [\tilde{b}_\alpha^l] - [\tilde{a}_\alpha^l] + [\tilde{1}_\alpha^l]$$

and so on.

In the fuzzy analysis of the factorial design, the treatment sum of squares is decomposed into orthogonal components corresponding to factorial effects, as given below:

$$\widetilde{SS^2_{A}}_\alpha^l = \frac{[\tilde{A}_\alpha^l]^2}{8r},\ \widetilde{SS^2_{B}}_\alpha^l = \frac{[\tilde{B}_\alpha^l]^2}{8r},\ \widetilde{SS^2_{AB}}_\alpha^l = \frac{[\widetilde{AB}_\alpha^l]^2}{8r},\ \widetilde{SS^2_{C}}_\alpha^l = \frac{[\tilde{C}_\alpha^l]^2}{8r}$$

$$\widetilde{SS^2_{AC}}_\alpha^l = \frac{[\widetilde{AC}_\alpha^l]^2}{8r},\ \widetilde{SS^2_{BC}}_\alpha^l = \frac{[\widetilde{BC}_\alpha^l]^2}{8r},\ \widetilde{SS^2_{ABC}}_\alpha^l = \frac{[\widetilde{ABC}_\alpha^l]^2}{8r}$$

The ANOVA table for lower-level model is given in Table 4.

3.3.2 Main effects and interactions in fuzzy 2^3 factorial design for the upper-level model

The same procedure is applied to the upper-level model using the upper bounds obtained from the α-cut

Table 4. Lower-level fuzzy ANOVA

Source of variation	Degree of freedom	Lower S.S.	Lower M.S.S.	Lower variance ratio 'F'
Replications	(r-1)	$\left(\widetilde{SS}^{2\,l}_{R_\alpha}\right)$	$\left(\widetilde{MSS}^{2\,l}_{R_\alpha}\right)=\frac{\widetilde{SS}^{2\,l}_{R_\alpha}}{r-1}$	$\widetilde{F}^{\,l}_{R_\alpha}=\frac{\left(\widetilde{MSS}^{2\,l}_{R_\alpha}\right)}{\left(\widetilde{MSS}^{2\,l}_{E_\alpha}\right)}\sim F\big(r-1,7(r-1)\big)$
$\widetilde{A}^l_\alpha$	1	$\left(\widetilde{SS}^{2\,l}_{A_\alpha}\right)$	$\left(\widetilde{MSS}^{2\,l}_{A_\alpha}\right)=\left(\widetilde{SS}^{2\,l}_{A_\alpha}\right)$	$\widetilde{F}^{\,l}_{A_\alpha}=\frac{\left(\widetilde{MSS}^{2\,l}_{A_\alpha}\right)}{\left(\widetilde{MSS}^{2\,l}_{E_\alpha}\right)}\sim F\big(r-1,7(r-1)\big)$
$\widetilde{B}^l_\alpha$	1	$\left(\widetilde{SS}^{2\,l}_{B_\alpha}\right)$	$\left(\widetilde{MSS}^{2\,l}_{B_\alpha}\right)=\left(\widetilde{SS}^{2\,l}_{B_\alpha}\right)$	$\widetilde{F}^{\,l}_{B_\alpha}=\frac{\left(\widetilde{MSS}^{2\,l}_{B_\alpha}\right)}{\left(\widetilde{MSS}^{2\,l}_{E_\alpha}\right)}\sim F\big(r-1,7(r-1)\big)$
$\widetilde{C}^l_\alpha$	1	$\left(\widetilde{SS}^{2\,l}_{C_\alpha}\right)$	$\left(\widetilde{MSS}^{2\,l}_{C_\alpha}\right)=\left(\widetilde{SS}^{2\,l}_{C_\alpha}\right)$	$\widetilde{F}^{\,l}_{C_\alpha}=\frac{\left(\widetilde{MSS}^{2\,l}_{C_\alpha}\right)}{\left(\widetilde{MSS}^{2\,l}_{E_\alpha}\right)}\sim F\big(r-1,7(r-1)\big)$
$\widetilde{AB}^l_\alpha$	1	$\left(\widetilde{SS}^{2\,l}_{AB_\alpha}\right)$	$\left(\widetilde{MSS}^{2\,l}_{AB_\alpha}\right)=\left(\widetilde{SS}^{2\,l}_{AB_\alpha}\right)$	$\widetilde{F}^{\,l}_{AB_\alpha}=\frac{\left(\widetilde{MSS}^{2\,l}_{AB_\alpha}\right)}{\left(\widetilde{MSS}^{2\,l}_{E_\alpha}\right)}\sim F\big(r-1,7(r-1)\big)$
$\widetilde{AC}^l_\alpha$	1	$\left(\widetilde{SS}^{2\,l}_{AC_\alpha}\right)$	$\left(\widetilde{MSS}^{2\,l}_{AC_\alpha}\right)=\left(\widetilde{SS}^{2\,l}_{AC_\alpha}\right)$	$\widetilde{F}^{\,l}_{AC_\alpha}=\frac{\left(\widetilde{MSS}^{2\,l}_{AC_\alpha}\right)}{\left(\widetilde{MSS}^{2\,l}_{E_\alpha}\right)}\sim F\big(r-1,7(r-1)\big)$
$\widetilde{BC}^l_\alpha$	1	$\left(\widetilde{SS}^{2\,l}_{BC_\alpha}\right)$	$\left(\widetilde{MSS}^{2\,l}_{BC_\alpha}\right)=\left(\widetilde{SS}^{2\,l}_{BC_\alpha}\right)$	$\widetilde{F}^{\,l}_{BC_\alpha}=\frac{\left(\widetilde{MSS}^{2\,l}_{BC_\alpha}\right)}{\left(\widetilde{MSS}^{2\,l}_{E_\alpha}\right)}\sim F\big(r-1,7(r-1)\big)$
$\widetilde{ABC}^l_\alpha$	1	$\left(\widetilde{SS}^{2\,l}_{ABC_\alpha}\right)$	$\left(\widetilde{MSS}^{2\,l}_{A_\alpha}\right)=\left(\widetilde{SS}^{2\,l}_{ABC_\alpha}\right)$	$\widetilde{F}^{\,l}_{ABC_\alpha}=\frac{\left(\widetilde{MSS}^{2\,l}_{ABC_\alpha}\right)}{\left(\widetilde{MSS}^{2\,l}_{E_\alpha}\right)}\sim F\big(r-1,7(r-1)\big)$
Error	7(r-1)	$\left(\widetilde{SS}^{2\,l}_{E_\alpha}\right)=$ By Subtraction	$\left(\widetilde{MSS}^{2\,l}_{E_\alpha}\right)=(\widetilde{SS}^{2\,l}_{E_\alpha})/(7(r-1)$	-
Total	8r-1	$\left(\sum\sum\sum\sum\widetilde{x^l_{ijkl_\alpha}}^{2}-\frac{\left(\sum\sum\sum\sum\widetilde{x^l_{ijkl_\alpha}}\right)^2}{N}\right)$		-

transformation. The ANOVA table is given in Table 5.

3.3.3 Fuzzy hypothesis and decision rule

In a fuzzy factorial design, the null and alternative hypotheses under the fuzzy statistics are that the factorial effects of the lower- and upper-level models are either significant or not. The following decision rule, shown in Figure 1, is used to check this.

If the calculated value of the F-statistic for the lower and upper levels is greater than the critical value of the F-statistic at the $\dot{\alpha}$ significance level, then the corresponding

Table 5. Upper-level fuzzy ANOVA

Source of variation	Degree of freedom	Upper S.S.	Upper M.S.S.	Upper variance ratio 'F'
Replications	(r-1)	$\left(\widetilde{SS^2_{R_\alpha}}^u\right)$	$\left(\widetilde{MSS^2_{R_\alpha}}^u\right)=\frac{\widetilde{SS^2_{R_\alpha}}^u}{r-1}$	$\widetilde{F_{R_\alpha}}^u=\frac{\left(\widetilde{MSS^2_{R_\alpha}}^u\right)}{\left(\widetilde{MSS^2_{E_\alpha}}^u\right)}\sim F(r-1,7(r-1))$
$\widetilde{A}$	1	$\left(\widetilde{SS^2_{A_\alpha}}^u\right)$	$\left(\widetilde{MSS^2_{A_\alpha}}^u\right)=\left(\widetilde{SS^2_{A_\alpha}}^u\right)$	$\widetilde{F_{A_\alpha}}^u=\frac{\left(\widetilde{MSS^2_{A_\alpha}}^u\right)}{\left(\widetilde{MSS^2_{E_\alpha}}^u\right)}\sim F(r-1,7(r-1))$
$\widetilde{B}$	1	$\left(\widetilde{SS^2_{B_\alpha}}^u\right)$	$\left(\widetilde{MSS^2_{B_\alpha}}^u\right)=\left(\widetilde{SS^2_{B_\alpha}}^u\right)$	$\widetilde{F_{B_\alpha}}^u=\frac{\left(\widetilde{MSS^2_{B_\alpha}}^u\right)}{\left(\widetilde{MSS^2_{E_\alpha}}^u\right)}\sim F(r-1,7(r-1))$
$\widetilde{C}$	1	$\left(\widetilde{SS^2_{C_\alpha}}^u\right)$	$\left(\widetilde{MSS^2_{C_\alpha}}^u\right)=\left(\widetilde{SS^2_{C_\alpha}}^u\right)$	$\widetilde{F_{C_\alpha}}^u=\frac{\left(\widetilde{MSS^2_{C_\alpha}}^u\right)}{\left(\widetilde{MSS^2_{E_\alpha}}^u\right)}\sim F(r-1,7(r-1))$
$\widetilde{AB}$	1	$\left(\widetilde{SS^2_{AB_\alpha}}^u\right)$	$\left(\widetilde{MSS^2_{AB_\alpha}}^u\right)=\left(\widetilde{SS^2_{AB_\alpha}}^u\right)$	$\widetilde{F_{AB_\alpha}}^u=\frac{\left(\widetilde{MSS^2_{AB_\alpha}}^u\right)}{\left(\widetilde{MSS^2_{E_\alpha}}^u\right)}\sim F(r-1,7(r-1))$
$\widetilde{AC}$	1	$\left(\widetilde{SS^2_{AC_\alpha}}^u\right)$	$\left(\widetilde{MSS^2_{AC_\alpha}}^u\right)=\left(\widetilde{SS^2_{AC_\alpha}}^u\right)$	$\widetilde{F_{AC_\alpha}}^u=\frac{\left(\widetilde{MSS^2_{AC_\alpha}}^u\right)}{\left(\widetilde{MSS^2_{E_\alpha}}^u\right)}\sim F(r-1,7(r-1))$
$\widetilde{BC}$	1	$\left(\widetilde{SS^2_{BC_\alpha}}^u\right)$	$\left(\widetilde{MSS^2_{BC_\alpha}}^u\right)=\left(\widetilde{SS^2_{BC_\alpha}}^u\right)$	$\widetilde{F_{BC_\alpha}}^u=\frac{\left(\widetilde{MSS^2_{BC_\alpha}}^u\right)}{\left(\widetilde{MSS^2_{E_\alpha}}^u\right)}\sim F(r-1,7(r-1))$
$\widetilde{ABC}$	1	$\left(\widetilde{SS^2_{ABC_\alpha}}^u\right)$	$\left(\widetilde{MSS^2_{A_\alpha}}^u\right)=\left(\widetilde{SS^2_{ABC_\alpha}}^u\right)$	$\widetilde{F_{ABC_\alpha}}^u=\frac{\left(\widetilde{MSS^2_{ABC_\alpha}}^u\right)}{\left(\widetilde{MSS^2_{E_\alpha}}^u\right)}\sim F(r-1,7(r-1))$
Error	7(r-1)	$\left(\widetilde{SS^2_{E_\alpha}}^u\right)=$ By Subtraction	$\left(\widetilde{MSS^2_{E_\alpha}}^u\right)=(\widetilde{SS^2_{E_\alpha}}^u)/(7(r-1)$	-
Total	8r-1	$\left(\widetilde{SS^2_{T_\alpha}}^u\right)=\left(\Sigma\Sigma\Sigma\Sigma\widetilde{x^u_{ijkl_\alpha}}^2-\frac{\left(\Sigma\Sigma\Sigma\Sigma\widetilde{x^u_{ijkl_\alpha}}\right)^2}{N}\right)$		-

individual treatment or combination of treatments is significant. Otherwise, it is not significant.

3.4 Algorithm for fuzzy factorial experiment analysis

To clearly describe the computational procedure of the proposed fuzzy factorial experiment framework, the analysis can be summarized in the following algorithmic steps.

Step 1. Represent each experimental observation as a triangular fuzzy number (p, q, r), where p, q, and r denote the lower, central, and upper values, respectively.

Step 2. Apply the α-cut transformation to convert the

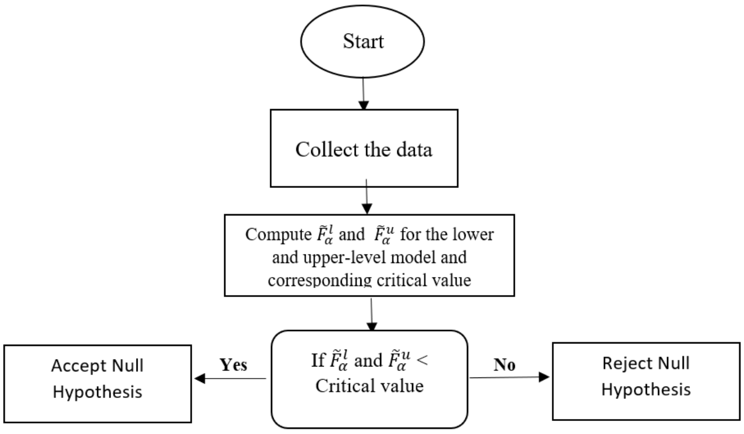

Figure 1. The test procedure for fuzzy 2^3 factorial experiment

triangular fuzzy numbers into interval-valued observations for a given $\alpha \in [0, 1]$.

Step 3. Obtain the lower-level dataset using the lower bounds of the α-cut intervals and construct the corresponding factorial experiment model.

Step 4. Obtain the upper-level dataset using the upper bounds of the α-cut intervals and construct the corresponding factorial experiment model.

Step 5. Compute the treatment totals and factorial effects using Yates' method for both the lower and upper models.

Step 6. Construct the ANOVA tables for the lower-level and upper-level factorial models.

Step 7. Compare the calculated F-statistics with the corresponding critical values to determine the significance of main and interaction effects.

Step 8. Interpret the results in the fuzzy environment by examining the range of outcomes across different values of α.

4. Illustration

In a physics experiment, the interaction effects of three factors – temperature (A), pressure (B), and magnetic field strength (C)—on the yield of a specific chemical reaction are examined. The dataset used in this illustration is synthetically constructed to demonstrate the proposed fuzzy factorial analysis method under uncertain observations. Because measurement instruments introduce uncertainty in the yield values, these measurements are represented as triangular fuzzy numbers to capture this imprecision. To analyze the main effects of temperature, pressure, and magnetic field strength on the reaction yield, a 2^3 factorial design replicated four times is used. The data is organized in a Randomized Block Design (RBD), and the observed yields are expressed as triangular fuzzy numbers. The factors are studied at two levels:

• Temperature (A): Low and High
• Pressure (B): Low and High
• Magnetic Field Strength (C): Low and High.

In the numerical illustration, the observed response values are represented as triangular fuzzy numbers (as given in Table 6) to incorporate measurement uncertainty present in the experimental process. The central value of each triangular fuzzy number represents the observed measurement, while the lower and upper bounds represent possible deviations due to instrument error or environmental variation. Using triangular fuzzy numbers provides a convenient way to represent uncertain observations. However, the proposed methodology is not limited to triangular fuzzy numbers and can also be applied to other fuzzy representations, such as trapezoidal fuzzy numbers, without altering the factorial analysis.

Analyze the 2^3 factorial design and determine whether the differences between the main effects A, B, and C are significant for the above fuzzy RBD at the 1% significance level.

After applying the α-cut method of defuzzification, we have the following lower-level values as give in Table 7.

Total number of observations is $8 \times 4 = 32$

$$[\tilde{G}^l_\alpha] = \text{Grand sum for lower-level fuzzy values}$$

$$= [998 + 62\alpha]$$

Total fuzzy sum of squares for lower level:

$$\left(\widetilde{SS}^{2\,l}_{T\alpha}\right) = \left(\{[25+2\alpha]^2 + [22+2\alpha]^2 + [26+2\alpha]^2 + \dots + [40+2\alpha]^2\} - \frac{[998+62\alpha]^2}{32}\right)$$

which simplifies to

$$\left(\widetilde{SS}^{2\,l}_{T\alpha}\right) = \left(\frac{35868+380\alpha^2-1064\alpha}{32}\right)$$

Table 6. Data for fuzzy RBD

Block I	Block II	Block III	Block IV
$\widetilde{ab}$ [34,36,38]	$\widetilde{bc}$ [27,29,32]	$\tilde{c}$ [32,34,36]	$\widetilde{ac}$ [35,39,41]
$\widetilde{bc}$ [26,28,31]	$\tilde{c}$ [30,32,34]	$\tilde{1}$ [26,28,29]	$\widetilde{ab}$ [31,32,33]
$\tilde{c}$ [35,37,39]	$\tilde{b}$ [24,27,28]	$\widetilde{ac}$ [24,26,29]	$\tilde{a}$ [41,42,44]
$\widetilde{ac}$ [21,22,24]	$\widetilde{ab}$ [33,35,37]	$\widetilde{bc}$ [33,34,36]	$\tilde{c}$ [39,41,43]
$\tilde{1}$ [25,27,29]	$\tilde{a}$ [34,36,39]	$\widetilde{ab}$ [31,33,35]	$\tilde{b}$ [31,33,34]
$\tilde{b}$ [29,31,33]	$\widetilde{abc}$ [30,32,33]	$\tilde{b}$ [27,29,31]	$\tilde{1}$ [21,23,24]
$\tilde{a}$ [40,41,43]	$\widetilde{ac}$ [39,41,44]	$\tilde{a}$ [30,32,33]	$\widetilde{abc}$ [40,42,45]
$\widetilde{abc}$ [43,44,46]	$\tilde{1}$ [22,24,26]	$\widetilde{abc}$ [36,39,41]	$\widetilde{bc}$ [31,33,35]

Table 7. Lower-level data value for fuzzy RBD after defuzzification

Treatment combination	Block I	Block II	Block III	Block IV	Treatment totals
$\tilde{1}^l_\alpha$	$[25 + 2\alpha]$	$[22 + 2\alpha]$	$[26 + 2\alpha]$	$[21 + 2\alpha]$	$[94 + 8\alpha]$
$\tilde{a}^l_\alpha$	$[40 + \alpha]$	$[34 + 2\alpha]$	$[30 + 2\alpha]$	$[41 + \alpha]$	$[145 + 6\alpha]$
$\tilde{b}^l_\alpha$	$[29 + 2\alpha]$	$[24 + 3\alpha]$	$[27 + 2\alpha]$	$[31 + 2\alpha]$	$[111 + 9\alpha]$
$\widetilde{ab}^l_\alpha$	$[34 + 2\alpha]$	$[33 + 2\alpha]$	$[31 + 2\alpha]$	$[31 + \alpha]$	$[129 + 7\alpha]$
$\tilde{c}^l_\alpha$	$[35 + 2\alpha]$	$[30 + 2\alpha]$	$[30 + 2\alpha]$	$[39 + 2\alpha]$	$[134 + 8\alpha]$
$\widetilde{ac}^l_\alpha$	$[21 + \alpha]$	$[39 + 2\alpha]$	$[24 + 2\alpha]$	$[35 + 4\alpha]$	$[119 + 9\alpha]$
$\widetilde{bc}^l_\alpha$	$[26 + 2\alpha]$	$[27 + 2\alpha]$	$[33 + \alpha]$	$[31 + 2\alpha]$	$[117 + 7\alpha]$
$\widetilde{abc}^l_\alpha$	$[43 + \alpha]$	$[30 + 2\alpha]$	$[36 + 3\alpha]$	$[40 + 2\alpha]$	$[149 + 8\alpha]$
Block total	$[253 + 13\alpha]$	$[239 + 17\alpha]$	$[237 + 16\alpha]$	$[269 + 16\alpha]$	$[998 + 62\alpha]$

Fuzzy sum of squares for blocks or replications for lower level:

$$\left(\widetilde{SS}^{2\,l}_{R_\alpha}\right) = \left(\frac{1}{8}\{[253 + 13\alpha]^2 + [239 + 17\alpha]^2 + [237 + 16\alpha]^2 + [269 + 16\alpha]^2\} - \frac{[998 + 62\alpha]^2}{32}\right)$$

which simplifies to

$$\left(\widetilde{SS}^{2\,l}_{R_\alpha}\right) = \left(\frac{2636+36\alpha^2-168\alpha}{32}\right)$$

Table 8. Fuzzy sum of square of treatments for lower-level model using Yates' algorithm

Lower-level treatment combinations (1)	Lower-level treatment total (2)	Treatment contrasts (3)	Effect contrasts (4)	Factorial effect estimates (5)	Fuzzy lower effects total	Fuzzy sum of squares
$\tilde{1}^l_\alpha$	$[94 + 8\alpha]$	$[239 + 14\alpha]$	$[479 + 30\alpha]$	$[998 + 62\alpha]$	$[\tilde{G}^l_\alpha]$	$\frac{[998+2\alpha]^2}{32}$
$\tilde{a}^l_\alpha$	$[145 + 6\alpha]$	$[240 + 16\alpha]$	$[519 + 32\alpha]$	$[86 - 2\alpha]$	$[\tilde{A}^l_\alpha]$	$\frac{[86-2\alpha]^2}{32}$
$\tilde{b}^l_\alpha$	$[111 + 9\alpha]$	$[253 + 17\alpha]$	$[69 - 4\alpha]$	[14]	$[\tilde{B}^l_\alpha]$	$\frac{[14]^2}{32}$
$\widetilde{ab}^l_\alpha$	$[129 + 7\alpha]$	$[266 + 15\alpha]$	$[17 + 2\alpha]$	[14]	$[\widetilde{AB}^l_\alpha]$	$\frac{[14]^2}{32}$
$\tilde{c}^l_\alpha$	$[134 + 8\alpha]$	$[51 - 2\alpha]$	$[1 + 2\alpha]$	$[40 + 2\alpha]$	$[\tilde{C}^l_\alpha]$	$\frac{[40+2\alpha]^2}{32}$
$\widetilde{ac}^l_\alpha$	$[119 + 9\alpha]$	$[18 - 2\alpha]$	$[13 - 2\alpha]$	$[-52 + 6\alpha]$	$[\widetilde{AC}^l_\alpha]$	$\frac{[-52+6\alpha]^2}{32}$
$\widetilde{bc}^l_\alpha$	$[117 + 7\alpha]$	$[-15 + \alpha]$	[-33]	$[12 - 4\alpha]$	$[\widetilde{BC}^l_\alpha]$	$\frac{[12-4\alpha]^2}{32}$
$\widetilde{abc}^l_\alpha$	$[149 + 8\alpha]$	$[32 + \alpha]$	[47]	[80]	$[\widetilde{ABC}^l_\alpha]$	$\frac{[80]^2}{32}$

Fuzzy sum of squares for treatments for lower level:

$$\left(\widetilde{SS^2_{Tr}}^l_\alpha\right) = \left(\frac{1}{4}\{[94 + 8\alpha]^2 + [145 + 6\alpha]^2 + [111 + 9\alpha]^2 + [129 + 7\alpha]^2 + \ldots + [149 + 8\alpha]^2\} - \frac{[998 + 62\alpha]^2}{32}\right)$$

$$\left(\widetilde{SS^2_{Tr}}^l_\alpha\right) = \left(\frac{18636+60\alpha^2-904\alpha}{32}\right)$$

Fuzzy sum of squares for errors:

$$\left(\widetilde{SS^2_E}^l_\alpha\right) = \widetilde{SS^2_T}^l_\alpha - \left(\widetilde{SS^2_{Tr}}^l_\alpha + \widetilde{SS^2_R}^l_\alpha\right)$$

$$\left(\widetilde{SS^2_E}^l_\alpha\right) = \left(\frac{35868 + 380\alpha^2 - 1064\alpha}{32}\right) - \left(\left(\frac{18636 + 60\alpha^2 - 904\alpha}{32}\right) + \left(\frac{2636 + 36\alpha^2 - 168\alpha}{32}\right)\right)$$

$$\left(\widetilde{SS^2_E}^l_\alpha\right) = \left(\frac{14596+284\alpha^2+8\alpha}{32}\right)$$

Using Yates' algorithm under a fuzzy environment, the breakdown of the fuzzy sum of squares of treatments for the lower-level model is shown in Table 8.

The fuzzy ANOVA table for the 2^3 factorial experiment under the lower-level model is shown in Table 9.

Between blocks for the lower-level model: The calculated $\widetilde{F_R}^l_\alpha$ values follow an F-distribution with 3 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 4.874. Hence, $\widetilde{F_R}^l_\alpha < F_t$ for all $0 \le \alpha \le 1$. We fail to reject the null hypothesis. There is no significant difference between the yields of the blocks.

Between treatments for the lower-level model: The

Table 9. ANOVA table for the 2^3 factorial experiment under the lower-level model

Source of variation	Degree of freedom	Lower S.S.	Lower M.S.S.	Lower variance ratio 'F'
Replications	3	$\left(\frac{2636+36\alpha^2-168\alpha}{32}\right)$	$\left(\frac{2636+36\alpha^2-168\alpha}{32*3}\right)$	$\frac{7*[2636+36\alpha^2-168\alpha]}{14596+284\alpha^2+8\alpha}$
Treatments	7	$\left(\frac{18636+60\alpha^2-904\alpha}{32}\right)$	$\left(\frac{18636+60\alpha^2-904\alpha}{32*7}\right)$	$\frac{3*[18636+60\alpha^2-904\alpha]}{14596+284\alpha^2+8\alpha}$
$\widetilde{A}_\alpha^l$	1	$\frac{[86-2\alpha]^2}{32}$	$\frac{[86-2\alpha]^2}{32}$	$\frac{21*[86-2\alpha]^2}{14596+284\alpha^2+8\alpha}$
$\widetilde{B}_\alpha^l$	1	$\frac{196}{32}$	$\frac{196}{32}$	$\frac{21*196}{14596+284\alpha^2+8\alpha}$
$\widetilde{C}_\alpha^l$	1	$\frac{[40+2\alpha]^2}{32}$	$\frac{[40+2\alpha]^2}{32}$	$\frac{21*[40+2\alpha]^2}{14596+284\alpha^2+8\alpha}$
$\widetilde{AB}_\alpha^l$	1	$\frac{196}{32}$	$\frac{196}{32}$	$\frac{21*196}{14596+284\alpha^2+8\alpha}$
$\widetilde{AC}_\alpha^l$	1	$\frac{[-52+6\alpha]^2}{32}$	$\frac{[-52+6\alpha]^2}{32}$	$\frac{21*[-52+6\alpha]^2}{14596+284\alpha^2+8\alpha}$
$\widetilde{BC}_\alpha^l$	1	$\frac{[12-4\alpha]^2}{32}$	$\frac{[12-4\alpha]^2}{32}$	$\frac{21*[12-4\alpha]^2}{14596+284\alpha^2+8\alpha}$
$\widetilde{ABC}_\alpha^l$	1	$\frac{6400}{32}$	$\frac{6400}{32}$	$\frac{21*6400}{14596+284\alpha^2+8\alpha}$
Error	21	$\left(\frac{14596+284\alpha^2+8\alpha}{32}\right)$	$\left(\frac{14596+284\alpha^2+8\alpha}{672}\right)$	-
Total	31	$\left(\frac{35868+380\alpha^2-1064\alpha}{32}\right)$	-	-

calculated $\widetilde{F_{Tr_\alpha}}^l$ values follow an F-distribution with 7 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 3.639. Hence, $\widetilde{F_{Tr_\alpha}}^l > F_t$ for all $0 \leq \alpha \leq 0.8$. There is a significant difference between the yields from treatment combinations applied in each block.

Main effect $\widetilde{A}_\alpha^l$ for the lower-level model: The calculated $\widetilde{F_{A_\alpha}}^l$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{A_\alpha}}^l > F_t$ for all $0 \leq \alpha \leq 1$.

Main effect $\widetilde{B}_\alpha^l$ for the lower-level model: The calculated $\widetilde{F_{B_\alpha}}^l$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{B_\alpha}}^l < F_t$ for all $0 \leq \alpha \leq 1$.

Main effect $\widetilde{C}_\alpha^l$ for the lower-level model: The calculated $\widetilde{F_{C_\alpha}}^l$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{C_\alpha}}^l < F_t$ for all $0 \leq \alpha \leq 1$.

Table 10. ANOVA table for the 2^3 factorial experiment under the upper-level model

Source of variation	Degree of freedom	Upper S.S.	Upper M.S.S.	Upper variance ratio 'F'
Replications	3	$\left(\frac{2051+27\alpha^2-426\alpha}{32}\right)$	$\left(\frac{2051+27\alpha^2-426\alpha}{32*3}\right)$	$\frac{7*[2051+27\alpha^2-426\alpha]}{[16165+349\alpha^2-774\alpha]}$
Treatments	7	$\left(\frac{19167+167\alpha^2-1542\alpha}{32}\right)$	$\left(\frac{19167+167\alpha^2-1542\alpha}{32*7}\right)$	$\frac{3*[19167+167\alpha^2-1542\alpha]}{[16165+349\alpha^2-774\alpha]}$
$\tilde{A}^u_\alpha$	1	$\frac{[85-\alpha]^2}{32}$	$\frac{[85-\alpha]^2}{32}$	$\frac{21*[85-\alpha]^2}{[16165+349\alpha^2-774\alpha]}$
$\tilde{B}^u_\alpha$	1	$\frac{[11+3\alpha]^2}{32}$	$\frac{[11+3\alpha]^2}{32}$	$\frac{21*[11+3\alpha]^2}{[16165+349\alpha^2-774\alpha]}$
$\tilde{C}^u_\alpha$	1	$\frac{[53-11\alpha]^2}{32}$	$\frac{[53-11\alpha]^2}{32}$	$\frac{21*[53-11\alpha]^2}{[16165+349\alpha^2-774\alpha]}$
$\widetilde{AB}^u_\alpha$	1	$\frac{[11+3\alpha]^2}{32}$	$\frac{[11+3\alpha]^2}{32}$	$\frac{21*[11+3\alpha]^2}{[16165+349\alpha^2-774\alpha]}$
$\widetilde{AC}^u_\alpha$	1	$\frac{[-51+5\alpha]^2}{32}$	$\frac{[-51+5\alpha]^2}{32}$	$\frac{21*[-51+5\alpha]^2}{[16165+349\alpha^2-774\alpha]}$
$\widetilde{BC}^u_\alpha$	1	$\frac{[7+\alpha]^2}{32}$	$\frac{[7+\alpha]^2}{32}$	$\frac{21*[7+\alpha]^2}{[16165+349\alpha^2-774\alpha]}$
$\widetilde{ABC}^u_\alpha$	1	$\frac{[79+\alpha]^2}{32}$	$\frac{[79+\alpha]^2}{32}$	$\frac{21*[79+\alpha]^2}{[16165+349\alpha^2-774\alpha]}$
Error	21	$\left(\frac{16165+349\alpha^2-774\alpha}{32}\right)$	$\left(\frac{16165+349\alpha^2-774\alpha}{672}\right)$	-
Total	31	$\left(\frac{35868+380\alpha^2-1064\alpha}{32}\right)$	-	-

Interaction effect $\widetilde{AB}^l_\alpha$ for the lower-level model: The calculated $\widetilde{F_{AB}}^l_\alpha$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{AB}}^l_\alpha < F_t$ for all $0 \le \alpha \le 1$.

Interaction effect $\widetilde{AC}^l_\alpha$ for the lower-level model: The calculated $\widetilde{F_{AC}}^l_\alpha$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{AC}}^l_\alpha < F_t$ for all $0 \le \alpha \le 1$.

Interaction effect $\widetilde{BC}^l_\alpha$ for the lower-level model: The calculated $\widetilde{F_{BC}}^l_\alpha$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{BC}}^l_\alpha < F_t$ for all $0 \le \alpha \le 1$.

Interaction effect $\widetilde{ABC}^l_\alpha$ for the lower-level model: The calculated $\widetilde{F_{ABC}}^l_\alpha$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{ABC}}^l_\alpha > F_t$ for all $0 \le \alpha \le 1$.

Table 11. Decisions based on lower-level and upper-level ANOVA tables using fuzzy logic

Source of variation	Lower-level model (significant between)	Upper-level model (significant between)	Overall (significant)
Block	-	-	-
Treatments	$0 \leq \alpha \leq 0.8$	$0 \leq \alpha \leq 0.8$	$0 \leq \alpha \leq 0.8$
$\widetilde{A}$	$0 \leq \alpha \leq 1$	$0 \leq \alpha \leq 1$	$0 \leq \alpha \leq 1$
$\widetilde{B}$	-	-	-
$\widetilde{AB}$	-	-	-
$\widetilde{C}$	-	-	-
$\widetilde{AC}$	-	-	-
$\widetilde{BC}$	-	-	-
$\widetilde{ABC}$	$0 \leq \alpha \leq 1$	$0 \leq \alpha \leq 1$	$0 \leq \alpha \leq 1$

Calculation for the upper-level model

After applying the same methodology to the upper bounds of the α-cuts, we obtain the following fuzzy ANOVA table (Table 10).

Between blocks for the upper-level model: The calculated $\widetilde{F_{R_\alpha}}^u$ values follow an F-distribution with 3 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 4.874. Hence, $\widetilde{F_{R_\alpha}}^u < F_t$ for all $0 \leq \alpha \leq 1$. We fail to reject the null hypothesis. There is no significant difference between the yields of the blocks.

Between treatments for the upper-level model: The calculated $\widetilde{F_{Tr_\alpha}}^u$ values follow an F-distribution with 7 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 3.639. Hence, $\widetilde{F_{Tr_\alpha}}^u > F_t$ for all $0 \leq \alpha \leq 0.8$. There is a significant difference between the yields from the treatment combinations applied to each block.

Main effect $\widetilde{A}_\alpha^u$ for the upper-level model: The calculated $\widetilde{F_{A_\alpha}}^u$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{A_\alpha}}^u > F_t$ for all $0 \leq \alpha \leq 1$.

Main effect $\widetilde{B}_\alpha^u$ for the upper-level model: The calculated $\widetilde{F_{B_\alpha}}^u$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{B_\alpha}}^u < F_t$ for all $0 \leq \alpha \leq 1$.

Main effect $\widetilde{C}_\alpha^u$ for the upper-level model: The calculated $\widetilde{F_{C_\alpha}}^u$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{C_\alpha}}^u < F_t$ for all $0 \leq \alpha \leq 1$.

Interaction effect $\widetilde{AB}_\alpha^u$ for the upper-level model: The calculated $\widetilde{F_{AB_\alpha}}^u$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{AB_\alpha}}^u < F_t$ for all $0 \leq \alpha \leq 1$.

Interaction effect $\widetilde{AC}_\alpha^u$ for the upper-level model: The calculated $\widetilde{F_{AC_\alpha}}^u$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{AC_\alpha}}^u < F_t$ for all $0 \leq \alpha \leq 1$.

Interaction effect $\widetilde{BC}_\alpha^u$ for the upper-level model: The calculated $\widetilde{F_{BC_\alpha}}^u$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at 0.01 significance level is 8.017. Hence, $\widetilde{F_{BC_\alpha}}^u < F_t$ for all $0 \leq \alpha \leq 1$.

Interaction effect $\widetilde{ABC}_\alpha^u$ for the upper-level model: The calculated $\widetilde{F_{ABC_\alpha}}^u$ values follow an F-distribution with 1 and 21 degrees of freedom. The tabulated value F_t at the 0.01 significance level is 8.017. Hence, $\widetilde{F_{ABC_\alpha}}^u > F_t$ for all $0 \leq \alpha \leq 1$.

As shown in Table 11, it can be concluded that there is no significant difference in chemical reaction yields across the different experimental blocks. However, the analysis shows significant differences in the effects of the individual factors (temperature, pressure, and magnetic field strength), as well as their interactions.

Of the eight possible combinations of the factors, temperature (A) is significant at every level of α. Additionally, the interaction of all three factors (temperature, pressure, and magnetic field strength) is significant for each $\alpha \in [0, 1]$, indicating that the combined effect of these factors plays a critical role in influencing the chemical reaction yield.

5. Comparison with classical factorial experiment

The proposed fuzzy factorial experiment framework extends classical factorial design methodology to situations where experimental observations involve uncertainty or vagueness. In classical factorial experiments, observations are assumed to be precise numerical values, and analysis is conducted using deterministic statistical models. However, in many practical applications, such as engineering experiments, environmental studies, and agricultural investigations, measurements may contain uncertainty due to instrument limitations, environmental variability, or observational imprecision.

In the proposed approach, experimental observations are represented using fuzzy numbers, which explicitly incorporate measurement uncertainty into the factorial analysis. By applying the α-cut transformation, fuzzy observations are converted into interval-valued data, resulting in lower and upper factorial models that are analyzed separately. This enables the evaluation of treatment effects over a range of possible values rather than relying on a single deterministic estimate.

Table 12. Comparison between the classical factorial design and the proposed fuzzy factorial design

Aspect	Classical factorial design	Proposed fuzzy factorial design
Data representation	Precise numerical observations	Observations represented as fuzzy numbers
Treatment of uncertainty	Not explicitly considered	Explicitly incorporated through fuzzy representation
Analytical output	Single deterministic estimate	Interval-based results reflecting uncertainty
Interpretation of results	Deterministic interpretation of factor effects	Uncertainty-aware interpretation based on lower and upper bounds
Relationship between models	-	Classical model obtained as a special case when fuzziness approaches zero

For a specific value of α, the classical factorial experiment model commonly used in traditional experimental designs can be considered a special case of the proposed fuzzy factorial framework. As the degree of fuzziness approaches zero, the lower and upper bounds of the fuzzy numbers converge to the same crisp value, and the fuzzy factorial model becomes the classical factorial model. Thus, the proposed framework captures the full range of possible values between the lower and upper bounds of fuzzy numbers, providing a more comprehensive representation of uncertainty in experimental observations. This enables a more reliable interpretation of treatment effects by accounting for variability in measurements arising from uncertainties in physical parameters such as temperature, pressure, and magnetic field strength. The conceptual differences between the classical factorial experiment framework and the proposed fuzzy factorial approach are summarized in Table 12.

6. Discussion

The results demonstrate that the proposed fuzzy factorial experiment framework offers a useful approach for analyzing experimental data when observations contain uncertainty. By representing observations as fuzzy numbers, the model incorporates experimental variability and measurement imprecision directly into the analysis. The α-cut based transformation enables the use of classical factorial analysis while preserving the uncertainty in the data. As a result, the analysis yields a range of possible treatment effects rather than a single deterministic estimate, providing a more informative basis for decision-making under uncertain conditions. However, some limitations should be noted. The computational effort may increase as the number of factors and replications grows, especially when both lower and upper models must be analyzed. In addition, the interpretation of results may depend on the chosen fuzzy representation for modeling the observations. Despite these limitations, the proposed framework offers a flexible extension of classical factorial analysis and may be useful in experimental situations where uncertainty in observations cannot be ignored. Future work may extend the approach to factorial experiments with more factors and levels, as well as explore alternative fuzzy representations for modeling uncertainty.

7. Conclusion

This paper proposes a fuzzy factorial experiment framework for a 2^3 design to analyze experimental data in the presence of uncertainty. Unlike classical factorial experiments, which assume precise observations, the proposed methodology uses triangular fuzzy numbers to represent imprecise measurements. The α-cut defuzzification approach converts fuzzy observations into interval-valued data, enabling factorial analysis through lower-level and upper-level models. Results from the numerical illustration demonstrate that the proposed fuzzy factorial design captures the variability in experimental observations and provides a more flexible interpretation of treatment effects under uncertain conditions. By analyzing both the lower and upper bounds of the fuzzy data, the proposed model offers a range of possible outcomes, reducing the risk of misleading conclusions that may result from ignoring measurement uncertainty. The proposed methodology is particularly useful in experimental situations where measurements are affected by instrument limitations, environmental variability, or subjective evaluations. Potential applications include engineering experiments, agricultural field trials, environmental studies, and industrial process optimization, where uncertain observations are common. Future research may extend the proposed fuzzy factorial framework to designs with more factors and levels, as well as explore other fuzzy representations, such as trapezoidal or interval-valued fuzzy numbers.

Authors' contribution

Rahul Thakur: Conceptualization, data curation, formal analysis, methodology, writing – original draft. Masum Raj: Validation, investigation, writing – review & editing. S.C. Malik: Supervision, validation, writing – review & editing.

Conflicts of interest

The authors declare that there are no known financial or personal conflicts of interest that could have influenced the research findings or conclusions presented in this study.

Data availability statement

No data was utilized in the research presented in this article.

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