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Original Research

Novel Russell's approximation method for transportation problem under hesitant bifuzzy environment

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Abstract: Transportation problem is one of the significant linear programming problems that emerges in several conditions and gains attention of many researchers. Reducing a commodity's transportation expense to meet demand at destination is the goal of the transportation problem. However, uncertainty plays a vital role in real life, making it challenging for decision-makers to provide precise values for the coefficients related to the transportation problem. Hesitant bifuzzy sets are a notable advancement of fuzzy set theory, as they allow decision-makers to deal with every hesitancy without any restriction. This study aims to introduce a formulation of transportation problem under a hesitant bifuzzy environment. In this study, we have introduced a novel Russel approximation method (RAM) for the hesitant bifuzzy transportation problem, where every parameter is represented by hesitant bifuzzy elements. To show the applicability of the proposed method, a real-life illustration has been taken in this article. Additionally, to demonstrate the superiority of the proposed method, a comparative study has also been conducted with other existing methods, which results in the minimum transportation cost.

Keywords: RAM, Hesitant bifuzzy Set, Fuzzy set, Transportation problem

1. Introduction

Linear programming problems have a significant impact on our daily lives. Transportation problems (TP) are among the most important linear programming problems. They play a significant role in developing strong client relationships in a skillful manner. Transportation problems encounter the challenges of delivering reliable, secure, and efficient transportation while reducing the negative effects on the environment and local populations. The primary goal of TP is to reduce transportation costs from sources to multiple destinations. TP was primarily developed in 1941 by Hitchcock [1]. Thereafter, Dantzig [2] conducted research and provided various methods to solve TP. Several methods have been developed to obtain an Initial Basic Feasible Solution (IBFS) namely, North-West corner, least cost method, Raw-minima method, Column-minima method, and Vogel's Approximation Method (VAM). The modified distribution (MODI) method and Stepping stone method are used to determine the most efficient transportation cost. In everyday life, optimization tasks are often imprecise and vague in nature. To deal with vagueness and uncertainty in our daily lives, Zadeh [3] introduced the concept of fuzzy sets. Since then, many extensions have been made by Atanassov [4,5], Chaube et al. [6], Torra [7], and Zhu et al. [8].

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Fuzzy transportation problems (FTP) are problems in which the parameters of transportation are given by fuzzy numbers. It has been predicted that fuzzy linear programming would be more consistent, as it can deal with uncertainty and vagueness in an efficient manner. Zimmermann [9] developed the fuzzy optimisation method to solve TP. Afterwards, Oheigeartaigh [10] suggested a method for addressing TP where supply and demand were given as fuzzy numbers. Later in 1984, Chanas et al. [11] invented a fuzzy linear programming model with crisp co-efficient and fuzzy supply and demand values. Then Chanas and Kuchta [12] proposed a technique to solve TP in which the terms of fuzzy coefficient were expressed as fuzzy numbers. And later Kaur & Kumar [13] also suggested an algorithm to address FTP. Afterwards, the fuzzy transportation problem has gained the attention of many researchers, and several methods to generalize fuzzy numbers and to solve transportation problems in various methods have been developed (Gupta et al. [14,15]; Liu & Kao [16]; Pandian & Natarajan [17]; Singh [18]. Several methods were proposed to solve transportation problems under different fuzzy environments (Pandian & Natarajan [17]; Singh [18]). In 2010, Pandian and Natarajan [17] introduced the zero-point method to solve FTP. The Russell's Approximation Method (RAM) was introduced by Hasibuan [19] for solving basic TP. Since, the RAM for the hesitant bifuzzy transportation problem (HBFTP) has not yet been developed in the literature, there is a need to develop a novel Russel's Approximation Method (RAM) for getting an initial basic feasible solution (IBFS) for hesitant bifuzzy TP.

In this work, we develop methods for HBFTP by taking into account HBFNs. The main contributions of this study are as follows:

- A novel RAM-based approach has been developed to improve the initial basic feasible solution (IBFS) of transportation problems.
- An algorithm for the Generalized Hesitant Bifuzzy Modified Distribution Method has been formulated.
- A practical real-life case study has been conducted to demonstrate the applicability, feasibility, and effectiveness of the proposed RAM and Generalized Hesitant Bifuzzy MODI techniques in solving transportation-related problems.
- A comparative analysis with existing methods confirms the superior robustness and performance of the developed approach, establishing its potential for broader use in optimization and transportation research.

The remainder of the article is structured as follows: primary definitions of HBFSs and the bifuzzy score function are given in section 2. In section 3, the problem formulation is defined. Section 4 presents the rules and steps of RAM for HBFTP. In section 5, a practical problem has been taken to demonstrate the relevance of the suggested method in a real-life situation. In section 6, a comparative study was conducted to show the superiority of the suggested method over other existing methods. Concluding remarks and future scope are given in section 7.

2. Preliminaries

The primary definition of hesitant bifuzzy set and the hesitant bifuzzy score function are given in this section.

2.1 The definition of fuzzy set by Zadeh [3]

If X is a collection of objects denoted by x, then a fuzzy set $\widetilde{A}$ in X is a set of ordered pair

$$\widetilde{A} = \{(x, \mu\widetilde{A}(\mathrm{x}) | \mathrm{x} \in \mathrm{X}\} \quad (1)$$

$\mu\widetilde{A}(\mathrm{x})$ is called the membership function (generalized characteristic function) which maps X to the membership space and $\mu\widetilde{A}(\mathrm{x}) \epsilon [0,1]$.

2.2 The definition of hesitant fuzzy set by Torra [7]

On the universal set X, defined hesitant fuzzy set as:

$$\alpha = \{x, g(x): x \in X\} \quad (2)$$

where the sets $g(x)$ have value in [0,1], which signifies the possible membership value for a set and for degrees of $x \in X$.

2.3 The definition of hesitant bifuzzy set by Chaube et al. [6]

Definition 1. Let X be universe of discourse, a hesitant bifuzzy set defined on X is given by

$$H = \{\langle x, h_H(x), g_H(x)\rangle : x \in X \} \quad (3)$$

where $h_H(x)$ and $g_H(x) \in [0, 1]$, represents a set of possible membership degree (MD) and a possible non-membership degree (NMD) of $x \in X$ to the set H, respectively, with the conditions: $0 \leq \alpha, \beta \leq 1$, $0 \leq \alpha^+ + \beta^+ < 2$, where $\alpha \in h_H(x)$, $\beta \in g_H(x)$, $\alpha^+ \epsilon\, h_H^+(x) = \cup_{\alpha \in h_H(x)} max\, \{\alpha\}$ and $\beta^+ \epsilon\, g_H^+(x) = \cup_{\beta \in g_H(x)} max\, \{\beta\}$ for all $x \in X$. Here, $b_H(x) = (h_H(x), g_H(x))$ represents an HBFE denoted by $b_H = (h_H, g_H)$, with the condition $\alpha \epsilon\, h_H$, $\beta \epsilon\, g_H$, $\alpha^+ \epsilon\, h_H^+(x) = \cup_{\alpha \in h_H(x)} max\, \{\alpha\}$, and $\beta^+ \epsilon\, g_H^+(x) = \cup_{\beta \in g_H(x)} max\, \{\beta\}$, $0 \leq \alpha, \beta \leq 1$ and $0 \leq \alpha^+ + \beta^+ < 2$.

2.4 The definition of bifuzzy score function by

Chaube et al. [6]

Definition 2. Let $B = \{\langle x, h_H(x), g_H(x)\rangle : x \in X\}$ be a hesitant bifuzzy set. The bifuzzy score function is given by

$$S_{hbfs}(x_i) = \left[\left(\frac{1}{\#h}\sum_{j=1}^{\#h} h_H^{\sigma(j)}(x_i)\right), \left(\frac{1}{\#g}\sum_{j=1}^{\#g} g_H^{\sigma(j)}(x_i)\right)\right] \quad (4)$$

Where, $\#h$ and $\#g$ are the total number of components in the MD and NMD set respectively.

With the help of bifuzzy score function, we converted hesitant bifuzzy element (HBFE) into a conflicting bifuzzy element (CBFE). After aggregating the HBFE and converting into CBFE with the help of score function, we defuzzified the elements with S-operator [9].

The S-operator is given by

$$S(x) = \frac{\mu_x - \vartheta_x + 1}{2} \quad (5)$$

Where HBFN is in the form $\langle x, \mu x, \vartheta x \rangle$ of a CBFN.

3. Model formulation

Uncertainty plays a vital role in our daily life. Therefore, the exact amount of the transportation cost c_{ij} may not be known. Studying OR using HBFS is important in several ways. In reality, there are numerous fields to be studied under the HBF environment, such as inventory and sustainable supplier selection (Gupta & Joshi [20]). In this study, taking into account the incomplete data on transportation factors in TP, we have used HBFS to tackle the real-life situation. Since the customer bears the cost of transportation, it is necessary to keep the expense of transportation to a minimum. Hence, the hesitant bifuzzy cost (C_{ij}) is presented in order to create the mathematical model of the transportation problems.

The *HBFTP* is given as follows:

$$\text{Optimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij} \quad (6)$$

$$\text{Subject to, } \sum_{j=1}^{n} x_{ij} \le a_i \; (i = 1,2, \dots, m) \quad (7)$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \; (j = 1,2, \dots, n) \quad (8)$$

$$x_{ij} \ge 0 \; \forall i,j \quad (9)$$

Where, z denotes the objective function in Eq. (6) which was designed based on hesitant bifuzzy cost C_{ij} (i =1,2,…,m; j =1,2,…,n). C_{ij} is transportation cost for each unit of an item from *ith* origin to *jth* destination; a_i (i =1,2,…m) and b_j (j =1,2,…n) are interpreted as supply and demand respectively, and

$$\sum_{i=1}^{m} a_i \ge \sum_{j=1}^{n} b_j \quad (10)$$

is the feasibility condition.

The cost C_{ij} is usually considered to be essentially deterministic. According to real life situation, we designed the problem based on hesitant bifuzzy sets. We took the hesitant bifuzzy cost as (h_{Bij}, g_{Bij}) for a corresponding route. In order to reduce the cost of transportation, DMs need to increase the MD h_{Bij} and decrease the NMD g_{Bij}.

4. Solution procedure of HBFTP by Russell's approximation method

In this section, we discussed the solution procedure of HBFTP with the help of Russell's Approximation Method (RAM).

4.1 Algorithm for Russell's approximation method

Step 1: Construct a hesitant bifuzzy cost function C_{ij} by taking into account the routes' priority and the possibility of delivering goods along these routes. Convert the hesitant bifuzzy cost function into a crisp cost using the bifuzzy score function.

Step 2: Check whether the problem is balanced, i.e., whether demand is equal to supply. If they are equal, proceed to the next step. If not, then we have to introduce a dummy row or column to balance the problem.

Step 3: Find the highest cost (u_i) of each row.

Step 4: Find the highest cost (v_j) of the column.

Step 5: For each variable in transportation problem obtain $\Delta_{ij} = C_{ij} - (u_i + v_j)$.

Step 6: Choose the variable that has the highest negative value of Δ and allocate as much as possible according to demand and supply. When either demand or supply becomes zero after allocation, cancel that row or column.

Step 7: Repeat steps 3 to 6 until all demand and supply goes to zero.

Step 8: Then the initial hesitant bifuzzy basic feasible solution (IHBBFS) C_{ij} and basic hesitant bifuzzy cost is $\sum C_{ij}.m_{ij}$.

The algorithm for Russell's approximation method is presented in Figure 1.

4.2 Generalized the hesitant bifuzzy modified distribution method

The hesitant bifuzzy modified distribution method is introduced in this section to obtain the optimal outcome of hesitant bifuzzy TP from IHBBFS by using the RAM.

Step 1: Obtain IHBBFS of HBTP by using the RAM.

Step 2: Then write the HBF variables u_i and v_j in

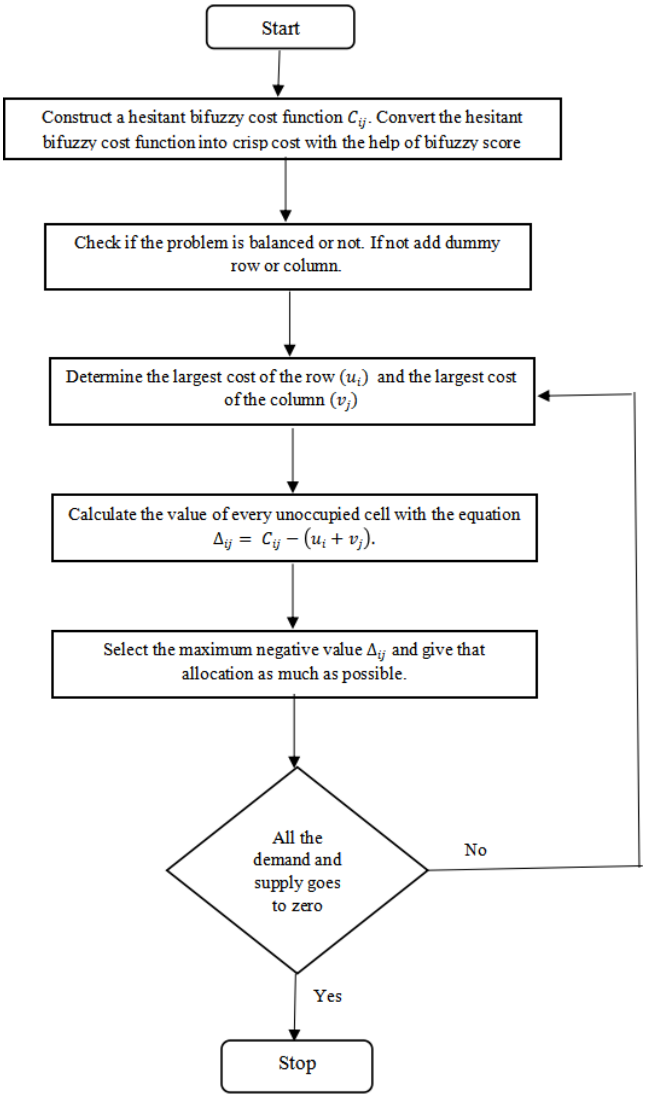

Figure 1. Algorithm for Russell's approximation method

accordance with each row and column. At the head of each row, write u_i, and at the bottom of each column, write v_j, respectively.

Step 3: Set any u_i or v_j to be zero where there is the maximum number of allocations.

Step 4: Then calculate the $c_{ij} = u_i + v_j$ (for every i and j) for each basic cell. From this, we can determine the values of u_i and v_j.

Step 5: For each non-basic cell, calculate $d_{ij} = C_{ij} - (u_i + v_j)$ (for every i and j, respectively). Then the following cases may arise:

Step 5.1: If every $d_{ij} \geq 0 \ \forall\ i,j$, then the obtained solution is optimal.

Step 5.2: If any $d_{ij} < 0$, then the resultant answer is not the optimal one. In this case, proceed to step 6.

Step 6: In the IHBBFS, choose d_{ij} which has the most negative score value. Let it be at cell (r,s), where $1 \leq r \leq m$ and $1 \leq s \leq n$.

Step 7: After that, choose a quantity θ in the (r, s) cell with the most negative d_{ij}, then make a loop:

Rule to define the loop: Starting with cell (r,s), allocate the amount θ horizontally or vertically and move to the next basic cell, with the restriction that the entire loop cannot be contained in a non-basic cell (except for the θ cell). In this way, we reach the θ cells and complete the loop.

Step 8: In the (r,s) cell, add the positive amount θ at the start of the loop, with alternating positive and negative signs at all endpoints to satisfy supply and demand constraints. Then, take the minimum value of θ that will make non-negative values in the revised solution for each basic variable.

Step 9: Insert the new value of θ and get the new allocation for the transportation table. When the value of θ is inserted, a cell assumes a value of 0. The cell with a value of 0 will become non-basic. This improves the HBFBFS. Now again, set one row or column to be zero and calculate the value of $C_{ij} = u_i + v_j$.

Step 10: Now repeat step 5. Calculate the value of $d_{ij} = C_{ij} - (u_i + v_j)$ for new transportation table until all $d_{ij} \geq 0$ $\forall\, i,j$.

Step 11: The obtained hesitant bifuzzy optimal solution is x_{ij} , and the hesitant bifuzzy optimal transportation cost is $\sum C_{ij} . x_{ij}$ $\forall\, i,j$.

5. Practical application of the hesitant bifuzzy transportation problem

A well-known business has three plants of electric appliances namely S_1(Bulb), S_2(Tube), and S_3(Fan) and the electrical appliances are distributed to three markets D_1(Dehradun), D_2(Delhi), and D_3(Haridwar), respectively. There are different routes to transport the appliances from the factories to the markets. Different transportation costs are assigned to each route. It is notified that customers prefer to minimize transportation costs. So, it is impossible for supplier to deliver materials to all customers due to vehicle capacity constraints. Consequently, hesitant bifuzzy sets are used to select the route. The company's primary goal is to reduce shipping costs through a TP under a hesitant bifuzzy environment.

Table 1 denotes the availability a_i (i =1,2,3) of the electrical appliances at the three sources S_i(i=1,2,3) and the demands b_j (j =1,2,3) at the three destinations D_j (j = 1,2,3). Now we determine the HBFTP and want to find out the optimal solution. According to the stepped algorithm, we first convert the hesitant bifuzzy number to crisp number by using the bifuzzy score function. Then, we use the RAM method to find the IBFS of the HBFTP.

Table 1. Formulating the hesitant bifuzzy cost

	D_1	D_2	D_3	a_i
S_1	$\langle$ {0.6,0.7,0.3} {0.2,0.9,0.3} $\rangle$ {22,32,40}	$\langle$ {0.8,0.5,0.4,0.2} {0.4,0.2,0.5,0.7} $\rangle$ {15,18,22,37}	$\langle$ {0.7,0.5,0.3} {0.3,0.7,0.8} $\rangle$ {23,26,38}	22
S_2	$\langle$ {0.5,0.4,0.3} {0.7,0.6,0.3} $\rangle$ {22,27,28}	$\langle$ {0.8,0.7,0.4} {0.2,0.4,0.8} $\rangle$ {32,38,42}	$\langle$ {0.7,0.6,0.2} {0.3,0.6,0.8} $\rangle$ {22,28,32}	30
S_3	$\langle$ {0.7,0.3,0.5} {0.3,0.6,0.5} $\rangle$ {32,34,42}	$\langle$ {0.3,0.2,0.8} {0.6,0.7,0.1} $\rangle$ {26,28,32}	$\langle$ {0.7,0.6,0.4,0.2} {0.3,0.9,0.5,0.2} $\rangle$ {33,37,32,38}	44
b_j	36	38	22	96

Step 1: Convert the HBFTP into crisp values by using bifuzzy score function given in Eq. (4) and Eq. (5). Table 2 contains the HBFTP converted into crisp values.

Let $B = \{\langle x, h_H(x), g_H(x)\colon x \in X\}$ be a hesitant bifuzzy set. The bifuzzy score function is given by

$$S_{hbfs}(x_i) = \left[\left(\frac{1}{\#h}\sum_{j=1}^{\#h} h_H^{\sigma(j)}(x_i)\right), \left(\frac{1}{\#g}\sum_{j=1}^{\#g} g_H^{\sigma(j)}(x_i)\right)\right] \quad (11)$$

Here, $\#h$ and $\#g$ are the total number of components in the MD and NMD sets, respectively.

The HBFTP values in Table 1 are converted to fuzzy cost ranking values as follows:

$$S_{hbfs}(x_i) = \left[\left(\frac{1}{\#h}\sum_{j=1}^{\#h} h_H^{\sigma(j)}(x_i)\right), \left(\frac{1}{\#g}\sum_{j=1}^{\#g} g_H^{\sigma(j)}(x_i)\right)\right]$$

$$= \left[\frac{1}{3}(0.6 + 0.7 + 0.3), \frac{1}{3}(0.2 + 0.9 + 0.3)\right]$$

$$= \left[\frac{1}{3}(1.6), \frac{1}{3}(1.4)\right]$$

$$= [0.533, 0.466]$$

Using $S(x) = \frac{\mu_x - \vartheta_x + 1}{2}$

$$S(x) = \frac{0.533 - 0.466 + 1}{2}$$

$$= 0.535$$

Step 2: The given HBFTP is balanced in nature.

Step 3: Find the highest cost u_i of each row.

Step 4: Find the highest cost v_j of each column.

The determined values of the row's highest cost (u_i) and the column's highest cost (v_j), as shown in Tables 3 and 4, are calculated as follows:

u_i = max (0.535, 0.512, 0.550) =0.550

v_j = max (0.535, 0.565, 0.520) =0.565

Step 5: Compute the cost of Δ_{ij} , where $\Delta_{ij} = C_{ij} - (u_i + v_j)$,

$\Delta_{11} = C_{11} - (u_{11} + v_{11})$ = 0.535− (0.550+0.565) = −0.580

$\Delta_{12} = C_{12} - (u_{12} + v_{12})$ = 0.512 − (0.550+0.585) = −0.623

$\Delta_{13} = C_{13} - (u_{13} + v_{13})$ = 0.550 − (0.550+0.565) = −0.565

$\Delta_{21} = C_{21} - (u_{21} + v_{21})$ = 0.565 − (0.585+0.565) = −0.585

$\Delta_{22} = C_{22} - (u_{22} + v_{22})$ = 0.585 − (0.585+0.585) = −0.585

$\Delta_{23} = C_{23} - (u_{23} + v_{23})$ = 0.530 − (0.585+0.550) = −0.605

$\Delta_{31} = C_{31} - (u_{31} + v_{31})$ = 0.520 − (0.565+0.520) = −0.565

$\Delta_{32} = C_{32} - (u_{32} + v_{32})$ = 0.515 − (0.585+0.520) = −0.590

$\Delta_{33}= C_{33} - (u_{33} + v_{33}) = 0.512 - (0.520+0.550) = -0.058$

The values of reduced cost to each Δ_{ij} are given in Table 5.

Step 6: Select the variable with the most negative Δ_{ij} and select the minimum demand and supply. Allocate to the cell with the lowest unit cost.

The most negative Δ_{ij} is (-0.623) in cell S_2D_2.

The most allocation to the cell is min (22,38) = 22.

This leaves the supply unchanged and completely meets the demand of 16 (38-22). The first allocation is given in Table 6.

Step 7: Eliminate row S_1 as demand becomes 0. Return to step 3 and repeat the procedure as many times as you can, ensuring the required cell is removed each time. The final allocations by RAM are given in Table 7.

$d_{11}=C_{11}-(u_1+v_1)=0.535-(0.520-0.003) =0.018$

$d_{13}=C_{21}-(u_1+v_3)=0.550-(0.485-0.003) =0.068$

$d_{22}=C_{22}-(u_2+v_2)=0.585-(0.515+0.045) =0.025$

$d_{33}=C_{33}-(u_3+v_3)=0.512-(0.485+0) =0.027$

Since all $d_{ij} \geq 0 \ \forall \ i,j$. Therefore, the obtained solution is optimal. The final allocations are $x_{12}= 22$, $x_{21}= 8$, $x_{23}= 22$, $x_{31}= 28$, and $x_{32}= 16$.

The minimum transportation cost is calculated as:

$Min =(22*23+16*28.66+22*27.66+8*25.66+28*36)$ $=2786.36$

The minimum allocated cells are $= m + n - 1 = 3+3-1 = 5$, therefore, it is a non-de-generate solution.

Table 2. The HBFTP converted into crisp values and fuzzy cost ranking value

	D_1	D_2	D_3	a_i	u_i
S_1	0.535 (31.33)	0.512 (23)	0.550 (29)	22	0.550
S_2	0.565 (25.66)	0.585 (37.33)	0.530 (27.66)	30	0.585
S_3	0.520 (36)	0.515 (28.66)	0.512 (35)	44	0.520
b_j	36	38	22		

Table 3. Determine u_i

	D_1	D_2	D_3	a_i	u_i
S_1	0.535	0.512	0.550	22	0.550
S_2	0.565	0.585	0.530	30	0.585
S_3	0.520	0.515	0.512	44	0.520
b_j	36	38	22		

Table 4. Determine v_j

	D_1	D_2	D_3	a_i
S_1	0.535	0.512	0.550	22
S_2	0.565	0.585	0.530	30
S_3	0.520	0.515	0.512	44
b_j	36	38	22	
v_j	0.565	0.585	0.550	

Table 5. Calculating the reduced cost to Δ_{ij}

	D_1	D_2	D_3	a_i	u_i
S_1	0.535 (−0.580)	0.512 (−0.623)	0.550 (−0.565)	22	0.550
S_2	0.565 (−0.585)	0.585 (−0.585)	0.530 (−0.605)	30	0.585
S_3	0.520 (−0.565)	0.515 (−0.590)	0.512 (−0.058)	44	0.520
b_j	36	38	22		
v_j	0.565	0.585	0.550		

Table 6. First allocation

	D_1	D_2	D_3	a_i
S_1	0.535	0.512 (22)	0.550	0
S_2	0.565	0.585	0.530	30
S_3	0.520	0.515	0.512	44
b_j	36	16	22	

Table 7. The final allocation

	D_1	D_2	D_3	a_i
S_1	0.535	0.512 (22)	0.550	22
S_2	0.565 (8)	0.585	0.530 (22)	30
S_3	0.520 (28)	0.515 (16)	0.512	44
b_j	36	16	22	

6. Comparison to the existing methods

The transportation problem is a common decision-making problem in daily life. The aim of TP is to minimize the transportation costs while meeting the constraints on supply and demand. It is very important to develop a model that can minimize the transportation costs. In literature, many TP have been studied, and various methods have been introduced to solve them. In order to confirm the superiority of our proposed method, a comparative analysis is carried out with other existing methods. We have summarized all transportation costs by different existing methods in Table 8.

From Table 8, we can conclude that the transportation cost obtained for HBFTP by RAM is the minimum. Table 8 validates the superiority of the developed method over other existing methods, as it gives the minimum optimal transportation cost. Additionally, the HBFIBFS obtained by RAM is also the minimum, therefore it can be used to obtain IBFS under a hesitant bifuzzy environment. The comparison to the existing methods is shown in Figure 2.

Table 8. Transportation cost by different methods

Methods	HBFIBFS	Optimal Transportation cost
North-West Corner method	3046.30	3046.30
Least Cost method	2789.36	2789.36
Vogels' Approximation method	2938.44	2786.36
Russel's Approximation method	2786.36	2786.36

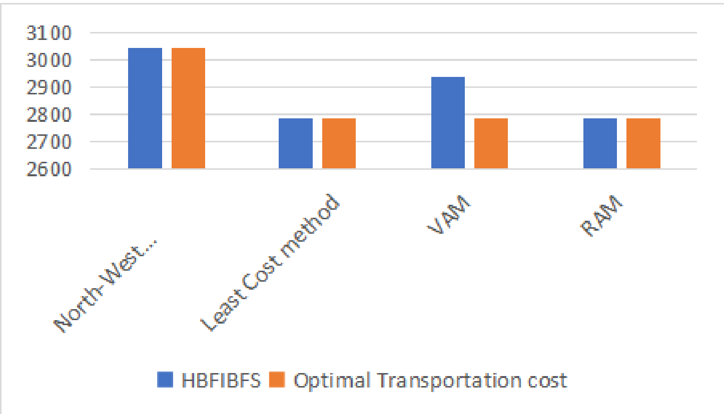

Figure 2. Comparison to the existing methods

7. Conclusion

Uncertainty has a significant impact on our daily lives; therefore, it is necessary to develop uncertain transportation problems that can address real-life decision-making problems. One such uncertain transportation problem is the fuzzy TP. Contrary to classical TPs under fuzziness, we have developed a new class of uncertain TPs, namely HBFTPs. In this study, we defined TP under a hesitant bifuzzy environment and formulated its mathematical model. To obtain the IBFS for HBFTP, an algorithm for RAM has also been designed. An algorithm to obtain the optimal solution for HBFTP has also been developed in this article. The effectiveness of the proposed method and the underlying model has been demonstrated using a numerical example. To illustrate the advantages of the developed method, a comparative study with other existing methods has also been conducted. In the future, we may create the transportation models with fixed charges under a hesitant bifuzzy environment and solve them by using any other suitable approach. Our method may also be used to solve problems in supply chain management, inventory optimisation, and other operations research and economics-related fields.

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Authors' contribution

Ismat Beg: Conceptualization, methodology, validation and editing. Soniya Gupta: Conceptualization, methodology and writing – original draft. Arpa Ghosh: Conceptualization, methodology and writing – original draft. Dheeraj Kumar Joshi: Conceptualization, methodology, validation, supervision and writing – review & editing.

Conflict of interests

The authors declare that there is no conflict of interest.

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Availability of data and materials

The data sets generated and/or analysed during the current study are included in this article. And if more information is needed, it is available from the corresponding author on reasonable request.

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