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Original Research

Decision-making techniques based on aggregation operators and similarity measures under q-rung orthopair hesitant fuzzy connection numbers and their application

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Abstract: The concept of q-rung orthopair hesitant fuzzy set represents an advancement and extension of hesitant fuzzy sets, encompassing both fuzzy sets and q-rung orthopair fuzzy sets. q-rung orthopair hesitant fuzzy set characterizes a set of membership and non-membership grades within the interval [0, 1], which enhances its adaptability compared to existing methods. This flexibility proves invaluable in providing more insightful data about various objects. The primary objective of this research is to introduce a decision-making technique in the context of q-RHF using the theory of set pair analysis (SPA). q-RHFS effectively handles ambiguous data by incorporating membership and non-membership grades, while the connection number (CN) based on SPA theory manages the intricacies of uncertainty and certainty structures by relying on "identity", "discrepancy" and "contrary" grades. Building on the relationship between q-RHFS and the connection number of set pair analysis, a comprehensive framework known as q-rung hesitant fuzzy connection number set (*qHCNS*) is developed. This model not only addresses uncertainty, but also offers valuable insights. Furthermore, this research introduces similarity measures derived from *qHCN* and examines their advantages through illustrative examples. Additionally, a novel approach to decision modeling utilizing these measures applied to medical diagnosis is also introduced. The application of this established model contributes to an effective approach and demonstrated its soundness and efficiency. In addition, a detailed comparative study is conducted with the existing models and advantages of proposed model. The research concludes with a summary of the authors' findings, highlighting the consistency and effectiveness of their work.

Keywords: q-rung orthopair fuzzy sets, Hesitant fuzzy sets, SPA, Similarity measure, Decision making, Optimization, Medical diagnosis

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1. Introduction

1.1 q-rung orthopair Fuzzy sets

In 1965, Zadeh introduced a pivotal theory known as the fuzzy set [1]. Fuzzy sets encompass elements from the universal set, each of which is assigned a degree of membership within the range [0,1]. Numerous scholars have dedicated their efforts to this theory and have applied it in various fields. For instance, Sarfraz [2] harnessed the concept of fuzzy sets in the group decision-making process. Jin et al. [3] extended the scope of fuzzy sets and developed Aczel-Alsina aggregation operators for multi-attribute decision making. Zimmermann [4] also put forth an innovative approach for decision making in fuzzy environments. Alcantund et al. [5] introduced the concept of separable fuzzy soft sets, which enables decision making involving both positive and negative attributes. Ali et al. [6,7] developed a novel framework by building upon existing fuzzy models and employing aggregation operators to amalgamate information for decision systems. Torra [8] introduced the influential concept of generalized fuzzy sets, which he referred to as "hesitant fuzzy sets", wherein membership grades encompass a collection of elements within the range [0,1].

Yager [9] introduced an exceptionally robust concept based on Attanssove's intuitionistic sets, which he termed "q-rung orthopair fuzzy sets" (q-ROFSs). In this set, the characterization of membership and non-membership grades is extended in a comprehensive way. Ali [10] provided a new perspective on Yager's q-ROFSs concept. Furthermore, Peng et al. [11] developed a set of information measures built upon q-rung orthopair fuzzy sets. Khan et al. [12] delved deeper into the concept of measures for q-ROFSs. Wang [13] introduced generalized similarity measures utilizing cosine functions for q-ROFSs. Li et al. [14] further explored this idea, focusing on preference relations and their practical applications. Garg et al. [15] made significant contributions by extending the q-ROFS concept, developing powerful aggregation operators and demonstrating their utility in decision-making processes. Shaheen et al. [16] addressed a pivotal question in the literature: the importance of q-ROFSs and provided convincing arguments to support their significance. Oraya et al. [17] showed how this concept can be applied to assess the impact of delays on residential construction projects. Wang et al. [18] further expanded upon Yager's concept by introducing p,q-rung orthopair fuzzy sets and optimized this model. Farid et al. [19] established Aczel-Alsina aggregation operators for multi-criteria decision-making scenarios. Feng [20] explored a probabilistic approach for q-rung sets that helps in the selection of the most appropriate option. Razzaque et al. [21] defined mathematical rings within the q-rung fuzzy environment. Sarkar et al. [22] made a significant contribution by developing a combined framework encompassing q-rung orthopair fuzzy sets and weighted dual hesitant fuzzy sets and applying it to multi-attribute decision-making scenarios. Recently, Jabeen et al. [23] integrated various aggregation operators, including the Bonferroni aggregator and the Aczel-Alsina aggregator, into the q-rung orthopair fuzzy model, exemplifying its practical application.

1.2 Set pair analysis

Zhao et al. [24] introduced the concept of Set Pair Analysis (SPA) as a powerful tool to address data uncertainty. Recognizing the significance of this approach, Decai et al. [25] harnessed the number of connections within Set Pair Analysis to tackle uncertainties by techniques of network planning. Jiang et al. [26] employed this concept in a comprehensive evaluation for urban planning projects. Su et al. [27] further expanded the theory of Set Pair Analysis and applied it to assess the health of urban ecosystems. Ali et al. [28] innovatively combined intuitionistic hesitant fuzzy sets with the connection number from Set Pair Analysis and used this hybrid approach for ranking the best alternatives in multi-criteria decision-making scenarios. Zou et al. [29] utilized the variable fuzzy sets model of Set Pair Analysis and fuzzy AHP to calculate flood risk assessments. Chen et al. [30] focused on models for evaluating water ecological security based on multivariate connection numbers and Markov chains. Xiang et al. [31] conducted a comprehensive literature survey spanning from 1989 to 2020 to investigate the development, applications and challenges of Set Pair Analysis in environmental sciences. Zhao et al. [32] explored a novel application of Set Pair Analysis, especially in the identification of mine water sources using the AHP-entropy method. Shi et al. [33] applied this concept to pattern recognition. Pan et al. [34] developed a methodology to evaluate coordination in water resources management and to effectively address conflicts. Ma et al. [35] proposed a model grounded in Set Pair Analysis for comprehensive evaluation of information security in electric power. Yang et al. [36] enhanced the Set Pair Analysis model for urban water security assessment. Chen et al. [37] introduced a model for selecting the optimal evaluation schemes based on the framework of Set Pair Analysis. Finally, Zhou et al. [38] extended the use of Set Pair Analysis in the coupling model for health evaluations of the Huangchuan River.

1.3 Similarity measures

Similarity measures serve the purpose of quantifying the extent of similarity or likeness between two sets, facilitating an evaluation of how closely or comparably these sets align with respect to the attributes of their constituent elements. The choice of a particular similarity measure hinges upon the specific context and the inherent characteristics of the sets to be compared. Diverse similarity measures can be found in the literature, each with its unique applications and constraints. Kausar et al. [39] delved into cosine similarity measures and applied them to enhance the sustainability of solid waste management within the framework of cubic m-polar fuzz sets. Ozhu et al. [40] extended the notion of the dice similarity measure to single-valued neutrosophic type-2 hesitant fuzzy information and employed it in multi-attribute decision-making scenarios. Garg [41] also delved into dice similarity measures and generalized them with dummy mean operators for application in TOPSIS models. Mehmood et al. [42-43] developed cosine similarity measures and aggregation operators for IHFSs and trigonometric similarity measures tailored to bipolar complex fuzzy soft sets and harnessed their approach in pattern recognition and medical diagnosis. Ma et al. [44] adopted similarity measures for extracting potential routes in urban customized bus systems, relying on vehicle trajectory clustering. Jia et al. [45] explored the relationship between Pythagorean fuzzy sets and similarity measures in their research work. Jin et al. [46] extended the concept of picture fuzzy distance and similarity measures within the framework of complete lattices and presented their practical applications. Kirisci et al. [47] introduced innovative cosine similarity measures and distance metrics for the extension of fuzzy sets and explored preference techniques. Saqlain et al. [48] introduced a set of similarity and distance measures for intuitionistic fuzzy hypersoft sets and presented practical applications. Ali et al. [49] developed the concept of vector similarity measures for dual hesitant fuzzy linguistic term sets and explored their applications. Recently, Rahim et al. [50] enhanced the TOPSIS method with an improved cosine similarity measure tailored to Fermatean fuzzy sets. Ganie et al. [51] ventured into the realm of q-rung orthopair fuzzy sets and explored novel similarity measures and entropy metrics as well as their practical applications. Zhang et al. [52-53] applied the concept of exponential similarity measures within the domain of cubic sets with confidence neutrosophic number, particularly in fuzzy multi-valued scenarios, to develop a group decision-making model.

1.4 Motivation

Based on an extensive literature review and recognizing the significance of fuzzy sets, SPA, and similarity measures in various domains, we have drawn inspiration to establish a relationship and amalgamate these frameworks to address issues related to uncertainty more comprehensively. It is evident that the amalgamation of these frameworks has the potential to address challenges in a broader range of scenarios where individual structures may fall short. The contributions of this research are elaborated as follows:

- In this research, we introduced the concept of hesitant fuzzy sets of q-rung orthopair by merging q-rung orthopair fuzzy sets with hesitant fuzzy sets. Hesitant fuzzy sets of q-rung orthopair represent a more general and effective tool for handling ambiguity and uncertainty.
- We further enhanced the framework by integrating the concept of connection number from Set Pair Analysis into the q-rung orthopair hesitant fuzzy environment. This approach led to the development of a new theory known as q-rung orthopair hesitant fuzzy connection number *qHCN*.
- The paper also extended the concept of Jaccard similarity measures and dice similarity measures. We introduced weighted Jaccard similarity measures and weighted dice similarity measures while discussing their key properties.
- These innovative concepts and measures are applied to solve real-life problems, primarily focusing on assessing the degree of similarity. A detailed flowchart of the entire algorithm is presented, and practical applications demonstrate the effectiveness of the models we have developed.

The remainder of the paper is structured as follows: Section 2 provides the foundational definitions and ideas that underpin our proposed approach. In Section 3, we delved into the development of the concept of q-rung orthopair hesitant fuzzy connection numbers and outline fundamental operations related to the model. Section 4 delves into the similarity measures for the proposed model and elucidate their properties. Section 5 and Section 6 encompasses the application of the developed algorithm to counsel students based on the evaluation of their study subjects. Finally, Section 7 concludes the paper with the authors' findings and recommendations.

2. Preliminaries

In this section, a brief overview of the existing fuzzy models such as IHFSs, q-ROFSs, and Set Pair Analysis theory with some basic operations are discussed.

Definition 1: [42] An IHFS H on x is denoted by two mappings π and ω. It is indicated by following expression:

$$H = \{(k, \pi(k), \omega(k)) \mid k \in x\}$$

$\pi(k)$ and $\omega(k)$ are sets of certain values in [0, 1], representing the grades for membership and non-membership of the element $k \in x$ with the condition such that $0 \leq max(\pi(k)) + max(\omega(k)) \leq 1$.

Definition 2: [42] Score and accuracy function for IHFS $H = (\pi_H, \omega_H)$ are defined as:

$$S(H) = \frac{S(\pi_H) - S(\omega_H)}{2}, S(H) \in [-1, 1]$$

$$T(H) = \frac{S(\pi_H) + S(\omega_H)}{2}, T(H) \in [0, 1]$$

Where, $S(\pi_H) = \frac{sum\ of\ all\ elements\ in\ (\pi_H)}{order\ of\ (\pi_H)}$,

$$S(\omega_H) = \frac{sum\ of\ all\ elements\ in\ (\omega_H)}{order\ of\ (\omega_H)}$$

Definition 3: [42] For two IHFSs H_1 and H_2, $H_1 > H_2$ where " > " refers preferred to if any of the given condition fulfilled,

$$S(H_1) > S(H_2);$$
$$S(H_1) = S(H_2) \text{ and } T(H_1) > T(H_2);$$

Definition 4: [9] For a universal set x, A q-rung orthopair fuzzy set (q-ROFS) H over x is formulated as below.

$$H = (k, \pi_H(k), \omega_H(k) : k \in x)$$

where $\pi : x \to [0, 1]$ and $\omega: x \to [0, 1]$ are respectively the functions granted to assign the degrees of membership and non-membership such that

$$0 \leq (\pi_H(k))^B + (\omega_H(k))^B \leq 1, B \geq 1$$

Definition 5: The score function to rank the q-ROFN H = (π, ω) is defined as

$$V(H) = (\pi^B - \omega^B)$$

Also, an accuracy function is defined as

$$L(H) = (\pi^B + \omega^B)$$

It is evident that $-1 \leq V(H) \leq 1$ and $0 \leq L(H) \leq 1$.

Definition 6: [24] A connection number between two sets P and Q is defined and represented as,

$$Z = \left(\frac{L}{U}\right) + \left(\frac{M}{U}\right)i + \left(\frac{N}{U}\right)j$$
$$Z = E + Fi + G$$

Where U is the "total number of features" where L and M respectively denotes the "identity" and "contrary" features. $M = U\text{-}L\text{-}N$ is the "discrepancy" feature of sets P and Q. It is also signified in the following equations $E = L/U$, $F = M/U$ and $G = N/U$, which denote the identity, discrepancy and contrary degree, respectively. Clearly, $0 \leq E, F, G \leq 1$ and $E + F + G = 1$. Also, $i \in [-1,1]$ is coefficient of "discrepancy degree" and j is the coefficient of "contrary degree" and $j = -1$.

Definition 7: [28] The intuitionistic hesitant fuzzy CN set (IHFCNS) corresponding to IHFE $H = \{(k, \pi(k), \omega(k)) \mid k \in x\}$ is described as

$$Z_H = \{(k, E_H(k) + F_H(k)\, i) + G_H(k)\, j \mid k \in x\}$$

Where $E_{\mathcal{H}}(k) = \frac{1}{m}\sum_{l=1}^{m}(\{a_l(1-b_l)\})$,

$$F_{\mathcal{H}}(k) = \frac{1}{m}\sum_{l=1}^{m}(\{1 - a_l(1-b_l) - b_l(1-a_l)\})$$

$$\text{and } G_{\mathcal{H}}(k) = \frac{1}{m}\sum_{l=1}^{m}(\{b_l(1-a_l)\}).$$

Where $a_l \in \pi_H(k)$ and $b_l \in \omega_H(k)$ signified the "identity", "discrepancy" and "contrary" degrees, respectively.

3. A novel proposed model q-rung orthopair hesitant fuzzy connection number

In this section, we proposed a novel generalized structure by adding three different frameworks, q-rung orthopair fuzzy sets, hesitant fuzzy sets and the concept of connection number of set pair analysis. This attractive and powerful model is known as connection number of q-rung orthopair hesitant fuzzy. Moreover, some basic operations were also discussed.

Definition 8: An q-RHFS H on x is denoted by two mappings π and ω. It is indicated by following equation:

$$H = \{(k, \pi_H(k), \omega_H(k)) \mid k \in x\}$$

$\pi_H(k)$ and $\omega_H(k)$ are sets of certain values in [0, 1], representing the membership grades and non-membership grades of the element $k \in x$ with the condition such that

$$0 \leq max\left(\pi_H^B(k)\right) + max\left(\omega_H^B(k)\right) \leq 1,\ B \geq 1.$$

Definition 9: The q-rung orthopair hesitant fuzzy CN set ($qHCNS$) corresponding to q-RHFN $H = \{(k, \pi_H(k), \omega_H(k)) \mid k \in x\}$ is described as

$$Z_H = \{(k, E_H(k) + F_H(k)\, i) + G_H(k)\, j : k \in x\}$$

Where,

$$E_H(k) = \frac{1}{m}\sum_{l=1}^{m}\sqrt[B]{a_l^B(1-b_l^B)},$$

$$F_H(k) = \frac{1}{m}\sum_{l=1}^{m}\sqrt[B]{1 - a_l^B(1-b_l^B) - b_l^B(1-a_l^B)}$$

$$\text{and } G_H(k) = \frac{1}{m}\sum_{l=1}^{m}\sqrt[B]{b_l^B(1-a_l^B)}.$$

Where $a_l \in \pi_H(k)$, $b_l \in \omega_H(k)$ and $B \geq 1$ signified the "identity", "discrepancy" and "contrary" degrees, respectively.

Example 1: Let $H_1 = \{(0.8, 0.3), (0.4, 0.6)\}$ and $H_2 = \{(0.5, 0.3), (0.4, 0.1)\}$ be two q-ROHFNs and for $B = 6$, we have two $qHCNs$ such that,

$$Z_{H1} = \{0.0218 + 0.9740i + 0.0041j\}$$
$$Z_{H2} = \{0.0013 + 0.9983i + 0.0003j\}$$

Definition 10: For two $qHCNs$ $Z_1(k) = E_1(k) + F_1(k)i + G_1(k)j$ and $Z_2(k) = E_2(k) + F_2(k)i + G_2(k)j$, we have

$Z_1(k) = Z_2(k) \Leftrightarrow E_1(k) = E_2(k), F_1(k) = F_2(k), G_1(k) = G_2(k)$

$Z_1(k) \leq Z_2(k)$ *if* $E_1(k) \leq E_2(k)$, $G_1(k) \geq G_2(k)$.

$Z_1(k)^C = G_1(k) + F_1(k)i + E_1(k)j$ is the complement of the $qHCN$ $Z_1(k) = E_1(k) + F_1(k)i + G_1(k)j$

Definition 11: If $Z_1 = E_1 + F_1 i + G_1 j$ and $Z_2 = E_2 + F_2 i + G_2 j$ are two $qHCNs$, then

$$Z_1 \oplus Z_2 = (1 - (1-E_1)(1-E_2)) + ((F_1+G_1)(F_2+G_2) - F_1F_2)i + (F_1F_2)j$$
$$Z_1 \otimes Z_2 = E_1E_2 + (1 - (1-F_1)(1-F_2))i + ((E_1+G_1)(E_2+G_2) - E_1E_2)j$$
$$C_1{}^{\lambda} = a_1{}^{\lambda} + (1 - (1-b_1)^{\lambda})i + ((a_1+c_1)^{\lambda} - a_1^{\lambda})j \quad \lambda > 0.$$

and are also $qHCNs$.

4. Power aggregation operators for q-rung orthopair hesitant fuzzy connection number

In this section, we defined the aggregation operators to calculate *qHCNs* and developed a series of power aggregation operators for *qHCNs* such as power average *qHCNs* aggregation operators (*qHCPA*), power weighted average *qHCN* aggregation operators (*qHCPWA*), power ordered weighted average *qHCN* aggregation operators (*qHCPOW*), power geometric *qHCN* aggregation operators (*qHCPG*), power weighted geometric *qHCN* aggregation operators (*qHCPWG*), and power ordered weighted geometric *qHCN* aggregation operators (*qHCPOWG*). Furthermore, the characteristics and properties of these operators were also discussed.

4.1 Power average aggregation operators for q-rung orthopair hesitant fuzzy connection numbers

Definition 12: Suppose that Z_i (k) $(i \in N)$ is be the collection of q-RHCNs then power average operator will be a mapping *qHCPA*: $Z_i^n \rightarrow Z$ and it is defined as

$$qHCPA(Z_1, Z_2, \ldots Z_n) = \frac{\underset{i=1}{\overset{n}{\oplus}}.\ (1+T(Z_i))Z_i}{\sum_{i=1}^{n}(1+T(Z_i))}$$

$$= \left(1 - \prod_{i=1}^{n}\left(1-(E_i)^{\frac{(1+T(Z_i)))}{\sum_{i=1}^{n}(1+T(Z_i))}}\right) + \left(\prod_{i=1}^{n}(F_i+G_i)^{\frac{(1+T(Z_i))}{\sum_{i=1}^{n}(1+T(Z_i))}} - \prod_{i=1}^{n}(F_i)^{\frac{(1+T(Z_i))}{\sum_{i=1}^{n}(1+T(Z_i))}}\right) i + \left(\prod_{i=1}^{n}(F_i)^{\frac{(1+T(Z_i))}{\sum_{i=1}^{n}(1+T(Z_i))}}\right)\right)$$

wherever, $T(Z_i)$ is the *Sup* of i^{th} the biggest *qHCE* (Z_i) by all the other *qHCEs*, that is,

$$T(Z_i) = \sum_{\substack{j=1\\ j\neq i}}^{n} Sup(Z_i, Z_j)$$

here *Sup* (Z_i, Z_j) is the *Sup* for Z_i from Z_j, and it is calculated by,

$$Sup\ (Z_i, Z_j) = 1 - d\ (Z_i, Z_j)$$

The *Sup* fulfills the given characteristics:

$$Sup\ (Z_i, Z_j) \in [0,1]$$

$$Sup\ (Z_i, Z_j) = Sup\ (Z_j, Z_i)$$

$$Sup\ (Z_i, Z_j) \geq Sup\ (Z_s, Z_t), \text{ if } d\ (Z_i - Z_j) \geq d\ (Z_s - Z_t)$$

where d is distance.

The support (*Sup*) amount is a similarity indicator.

Definition 13: Let Z_i be a set of *qHCEs* and $w = (w_1, w_2, \ldots, w_n)^T$ is weight vector of Z_i, $w_i > 0$ and $\sum_{i=1}^{n} w_i = 1$. The power weight average operator *qHCPWA*: $Z^n \rightarrow Z$ is defined as

$$qHCPWA(Z_1, Z_2, \ldots, Z_n) = \frac{\underset{i=1}{\overset{n}{\oplus}}\ (w_i(1+T(Z_i)Z_i)}{\sum_{i=1}^{n} w_i(1+T(Z_i))}$$

$$= 1 - \prod_{i=1}^{n}\left(1-(E_i)^{\frac{w_i(1+T(Z_i)))}{\sum_{i=1}^{n} w_i(1+T(Z_i))}}\right) + \left(\prod_{i=1}^{n}(F_i+G_i)^{\frac{w_i(1+T(Z_i))}{\sum_{i=1}^{n} w_i(1+T(Z_i))}} - \prod_{i=1}^{n}(F_i)^{\frac{w_i(1+T(Z_i))}{\sum_{i=1}^{n} w_i(1+T(Z_i))}}\right) i + \left(\prod_{i=1}^{n}(F_i)^{\frac{w_i(1+T(Z_i))}{\sum_{i=1}^{n} w_i(1+T(Z_i))}}\right) j$$

The following properties can be easily justified for the *qHCPWA* operators.

$$\text{If } Z_i = Z \text{ then, } qHCPWA\,(Z_1, Z_2, \ldots, Z_n) = Z$$

$$Z^- \leq qHCPWA\ (Z_1, Z_2, \ldots, Z_n) \leq Z^+$$

$$\text{where } Z^- = \min_i Z_i \text{ and } Z^+ = \max_i Z_i$$

$$If\ Z_i \leq Z'_i \text{ then,}$$

$$qHCPWA(Z_1, Z_2, \ldots, Z_n) \leq qHCPWA\left(Z'_1, Z'_2, \ldots, Z'_n\right)$$

Definition 14: Let Z_i be a group of *qHCEs*, the power order weight average operator is a mapping *qHCPOWA: $Z^n \rightarrow Z$* and is defined as

$$qHCPOWA(Z_1, Z_2, \ldots, Z_n) \frac{\underset{i=1}{\overset{n}{\oplus}}\ (w_i(1+T(Z_{\sigma(i)})Z_{\sigma(i)})}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}$$

$$= 1 - \prod_{i=1}^{n}\left(1-(E_{\sigma(i)})^{\frac{w_i(1+T(Z_{\sigma(i)})))}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}}\right) + \left(\prod_{i=1}^{n}(F_{\sigma(i)}+G_{\sigma(i)})^{\frac{w_i(1+T(Z_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}} - \prod_{i=1}^{n}(F_{\sigma(i)})^{\frac{w_i(1+T(Z_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}}\right) i + \left(\prod_{i=1}^{n}(F_{\sigma(i)})^{\frac{w_i(1+T(Z_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}}\right) j$$

where $\sigma(1), \sigma(2), \ldots \sigma(n)$, indicates permutation of (1, 2, ... n), in which $Z_{\sigma(i-1)} \geq Z_{\sigma(i)}$, w_i $(i \in \mathrm{N})$ is group of weights so that

$$w_i = g\left(\frac{R_i}{TV}\right) - g\left(\frac{R_{i-1}}{TV}\right),\ R_i = \sum_{i=1}^{n} V_{\sigma(i)},$$

$$TV = \sum_{i=1}^{n} V_{\sigma(i)},\ V_{\sigma(i.)} = 1. + T\left(Z_{\sigma(i)}\right)$$

and $T(Z_{\sigma(j)})$ implies the *Sup* of *j*th largest *qHCE* $T(Z_{\sigma(j)})$ by all the other (*qHCEs*), that is,

$$T\left(Z_{\sigma(j)}\right) = \sum_{\substack{i=1\\ i\neq j}}^{n} Sup(Z_{\sigma(j)}, Z_{\sigma(i)})$$

where $\sum_{\substack{i=1\\ i\neq j}}^{n}$ *Sup* $(Z_{\sigma(j)}, Z_{\sigma(i)})$ shows the *Sup* of *j*th is the biggest *qHCE* $Z_{\sigma(j)}$, for the *i*th largest IHCE $Z_{\sigma(i)}$.

Some properties of *qHCPOWA* operator are as follows.

$$\text{If } Z_i = Z \text{ then } qHCPOWA(Z_1, Z_2, \dots, Z_n) = Z$$

$$Z^- \leq qHCPOWA(Z_1, Z_2, \dots, Z_n) \leq Z^+$$

$$\text{where } Z^- = \min_i Z_i \text{ and } Z^+ = \max_i Z_i$$

$$\text{if } Z_i \leq Z'_i \text{ then}$$

$$qHCPOWA(Z_1, Z_2, \dots, Z_n) = qHCPOWA(Z'_1, Z'_2, \dots, Z'_n)$$

Where Z'_j is a permutation of Z_j.

Definition 15: Suppose Z_i is a group of *qHCEs*, the power hybrid average operator *qHCPHA*: $Z^n \rightarrow Z$ is defined as

$$qHCPHA(Z_1, Z_2, \dots, Z_n) = \frac{\bigoplus_{i=1}^{n} (w_i(1+T(\dot{Z}_{\sigma(i)})\dot{Z}_{\sigma(i)})}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}$$

$$= 1 - \prod_{i=1}^{n}\left(1 - (\dot{E}_{\sigma(i)})^{\frac{w_i(1+T(\dot{Z}_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}}\right)$$

$$+ \left(\prod_{i=1}^{n}(\dot{F}_{\sigma(i)} + \dot{G}_{\sigma(i)})^{\frac{w_i(1+T(\dot{Z}_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}} - \prod_{i=1}^{n}(\dot{F}_{\sigma(i)})^{\frac{w_i(1+T(\dot{Z}_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}}\right) i$$

$$+ \left(\prod_{i=1}^{n}(\dot{F}_{\sigma(i)})^{\frac{w_i(1+T(\dot{Z}_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}}\right) j$$

Where $Z_{\sigma(i)}$ is the ith largest objects in *qHCE* arguments

$$\dot{Z}_i \, (\dot{Z} = (nw_i)Z_i \quad w = (w_1, w_2, \dots, w_n)$$

and it is the weighting vector of *qHCE* arguments Z_i (i = 1, 2, ..., n), $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$ and w_i is a group such that

$$w_i = g\left(\frac{R_i}{TV}\right) - g\left(\frac{R_{i-1}}{TV}\right),\ R_i = \sum_{i=1}^{j} V_{\sigma(i)},$$

$$TV = \sum_{i=1}^{n} V_{\sigma(i)},\ V_{\sigma(i)} = 1 + T(\dot{Z}_{\sigma(i)})$$

and $T(\dot{Z}_{\sigma(i)})$ is the *Sup* of jth biggest *qHCEs* $\dot{Z}_{\sigma(i)}$ by all the other (*qHCEs*), that is,

$$T(\dot{Z}_{\sigma(i)}) = \sum_{\substack{i=1 \\ i \neq j}}^{n} Sup(\dot{Z}_{\sigma(j)}, Z_{\sigma(i)})$$

Where $\sum_{\substack{i=1 \\ i\neq j}}^{n} Sup\,(\dot{Z}_{\sigma(j)}\ Z_{\sigma(i)})$ shows the *Sup* of jth biggest *qHCE* $\dot{Z}_{\sigma(j)}$ for the ith biggest *qHCE* $\dot{Z}_{\sigma(i)}$. Specifically, *qHCPHA* is reduced to *qHCPWA* operator if

$$w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^T$$

and *qHCPHA* is reduced to *IHCPOWA* operator if

$$w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)$$

4.2 Power geometric aggregation operators for q-rung orthopair hesitant fuzzy connection numbers

Definition 16: Suppose Z_i is family of *qHCEs*, power geometric operator *qHCPG*: $Z^n \rightarrow Z$ is defined as

$$qHCPG(Z_1, Z_2, \dots, Z_n) = \bigotimes_{i=1}^{n} (Z)^{\frac{1+T(Z_i)}{\sum_{i=1}^{n}(1+T(Z_i))}}$$

$$= \prod_{i=1}^{n}(E_i)^{\frac{(1+T(Z_i))}{\sum_{i=1}^{n}(1+T(Z_i))}} + \left(1 - \prod_{i=1}^{n}\left(1 - F_i\right)^{\frac{(1+T(Z_i))}{\sum_{i=1}^{n}(1+T(Z_i))}}\right) i$$

$$+ \left(\prod_{i=1}^{n}(E_i + G_i)^{\frac{(1+T(Z_i))}{\sum_{i=1}^{n}(1+T(Z_i))}} - \prod_{i=1}^{n}(E_i)^{\frac{(1+T(Z_i))}{\sum_{i=1}^{n}(1+T(Z_i))}}\right) j$$

where

$$T(Z_i) = \sum_{\substack{j=1 \\ i\neq j}}^{n} Sup(Z_i, Z_j)$$

Definition 17: Let Z_i be a group of *qHCEs* then power weight geometric operator *qHCPWG*: $Z^n \rightarrow Z$ is defined as

$$qHCPWG(Z_1, Z_2, \dots, Z_n) = \bigotimes_{i=1}^{n} (Z)^{\frac{w_i(1+T(Z_i))}{\sum_{i=1}^{n} w_i(1+T(Z_i))}}$$

$$= \prod_{i=1}^{n}(E_i)^{\frac{w_i(1+T(Z_i))}{\sum_{i=1}^{n} w_i(1+T(Z_i))}} + \left(1 - \prod_{i=1}^{n}\left(1 - F_i\right)^{\frac{w_i(1+T(Z_i))}{\sum_{i=1}^{n} w_i(1+T(Z_i))}}\right) i$$

$$+ \left(\prod_{i=1}^{n}(E_i + G_i)^{\frac{w_i(1+T(Z_i))}{\sum_{i=1}^{n} w_i(1+T(Z_i))}} - \prod_{i=1}^{n}(E_i)^{\frac{w_i(1+T(Z_i))}{\sum_{i=1}^{n} w_i(1+T(Z_i))}}\right) j$$

The following characteristics can be easily proved for *qHCPWG* operator.

$$\text{If } Z_i = Z \text{ then, } qHCPWG(Z_1, Z_2, \dots, Z_n) = Z$$

$$Z^- \leq qHCPWG(Z_1, Z_2, \dots, Z_n) \leq Z^+$$

$$\text{where } Z^- = \min_i Z_i \text{ and } Z^+ = \max_i Z_i$$

$$\text{If } Z_i \leq Z'_i \text{ then}$$

$$qHCPWG(Z_1, Z_2, \dots, Z_n) \leq qHCPWG(Z'_1, Z'_2, \dots, Z'_n)$$

Definition 18: Let Z_i is a group of *qHCEs*, the power order weight geometric operator of dimension 'n', *qHCPOWG*: $Z^n \rightarrow Z$ is defined as

$$qHCPOWG(Z_1, Z_2, \dots, Z_n) = \underset{i=1}{\overset{n}{\otimes}} \left(Z_{\sigma(i)}\right)^{\frac{w_i\left(1+T\left(Z_{\sigma(i)}\right)\right)}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}}$$

$$= \prod_{i=1}^{n}\left(\left(E_{\sigma(i)}\right)^{\frac{w_i(1+T(Z_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}}\right) + (1 - \prod_{i=1}^{n}\left(1 - F_{\sigma(i)}\right)^{\frac{w_i(1+T(Z_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}}) i + \left(\prod_{i=1}^{n}\left(E_{\sigma(i)} + G_{\sigma(i)}\right)^{\frac{w_i(1+T(Z_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}} - \prod_{i=1}^{n}\left(E_{\sigma(i)}\right)^{\frac{w_i(1+T(Z_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(Z_{\sigma(i)}))}}\right) j$$

where $\sigma(1), \sigma(2), \dots \sigma(n)$ indicates permutation of (1,2, …,), where $Z_{\sigma(i\text{-}1)} \geq Z_{\sigma(i)}$, w_i ($i = 1, 2, \dots n$) is a group of weights so that,

$$w_i = g\left(\frac{R_i}{TV}\right) - g\left(\frac{R_{i-1}}{TV}\right),\ R_j = \sum_{i=1}^{j} V_{\sigma(i)},$$

$$TV = \sum_{i=1}^{n} V_{\sigma(i)},\ V_{\sigma(i.)} = 1. + T\left(Z_{\sigma(i)}\right)$$

and $T(Z_{\sigma(i)})$ implies the *Sup* of *i*th biggest *qHCE* $Z_{\sigma(i)}$ by all the other *qHCEs*, that is,

$$T\left(Z_{\sigma(i)}\right) = \sum_{\substack{j=1 \\ i \neq j}}^{n} Sup(Z_{\sigma(i)}, Z_{\sigma(j)})$$

where $\sum_{\substack{j=1 \\ i\neq j}}^{n} Sup(Z_{\sigma(i)}, Z_{\sigma(j)})$ shows the *Sup* of *i*th is the biggest *qHCE* $Z_{\sigma(i)}$, for the *j*th largest *qHCE* $Z_{\sigma(j)}$.

Some properties of *qHCPOWG* operator are as follows,

$$\text{If } Z_i = Z \text{ then, } qHCPOWG(Z_1, Z_2, \dots, Z_n) = Z$$

$$Z^- \leq qHCPOWG(Z_1, Z_2, \dots, Z_n) \leq Z^+$$

$$\text{where } Z^- = \min_i Z_i \text{ and } Z^+ = \max_i Z_i$$

$$\text{If } Z_i \leq Z'_i \text{ then}$$

$$qHCPOWG(Z_1, Z_2, \dots, Z_n) \leq qHCPOWG\left(Z'_1, Z'_2, \dots, Z'_n\right)$$

$$\text{If } Z_i \leq Z'_i \text{ then}$$

$$qHCPOWG(Z_1, Z_2, \dots, Z_n) = qHCPOWG\left(Z'_1, Z'_2, \dots, Z'_n\right)$$

$$\text{Where } Z'_j \text{ be a permutation of } Z_j.$$

Definition 19: Suppose Z_i is a group of *qHCEs*, the power hybrid geometric operator of objects '*n*' *qHCPHG*: $Z^n \rightarrow Z$ is defined as

$$qHCPHG(Z_1, Z_2, \dots, Z_n) = \underset{i=1}{\overset{n}{\otimes}} (\dot{Z}_{\sigma(i)})^{\frac{w_i(1+T(\dot{Z}_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}}$$

$$= \prod_{i=1}^{n}(\dot{E}_{\sigma(i)})^{\frac{w_i(1+T(\dot{Z}_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}} + (1 - \prod_{i=1}^{n}\left(1 - \dot{F}_{\sigma(i)}\right)^{\frac{w_i(1+T(\dot{Z}_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}}) i + \left(\prod_{i=1}^{n}(\dot{E}_{\sigma(i)} + \dot{G}_{\sigma(i)})^{\frac{w_i(1+T(\dot{Z}_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}} - \prod_{i=1}^{n}(\dot{E}_{\sigma(i)})^{\frac{w_i(1+T(\dot{Z}_{\sigma(i)}))}{\sum_{i=1}^{n} w_i(1+T(\dot{Z}_{\sigma(i)}))}}\right) j$$

where $Z_{\sigma(i)}$ is the *i*th largest object in *qHCEs* arguments

$$\dot{Z}_i\ (\dot{Z} = (nw_i)Z_i,\ i = 1, 2, \dots, n),$$

$$w = (w_1, w_2, \dots, w_n)$$

and it is the weighting vector of IHCE influences Z_i as well as w_i is a group where,

$$w_i = g\left(\frac{R_i}{TV}\right) - g\left(\frac{R_{i-1}}{TV}\right),\ R_i = \sum_{i=1}^{j} V_{\sigma(i)},$$

$$TV = \sum_{i=1}^{n} V_{\sigma(i)},\ V_{\sigma(i)} = 1 + T(\dot{Z}_{\sigma(i)})$$

and $T(\dot{Z}_{\sigma(i)})$ is the *Sup* of *j*th biggest *qHCEs* $\dot{Z}_{\sigma(i)}$ by all the other (*qHCEs*), that is,

$$T\left(\dot{Z}_{\sigma(i)}\right) = \sum_{\substack{j=1 \\ i \neq j}}^{n} Sup(\dot{Z}_{\sigma(i)}, Z_{\sigma(j)})$$

where $\sum_{\substack{j=1 \\ i\neq j}}^{n} Sup(\dot{Z}_{\sigma(i)}, Z_{\sigma(j)})$ shows the *Sup* of *j*th largest *qHCE* $\dot{Z}_{\sigma(j)}$, for the *i*th biggest *qHCE* $\dot{Z}_{\sigma(i)}$. Specifically, *qHCPHG* is reduced to *qHCPWG* operator if

$$w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^T$$

and *qHCPHG* is reduced to operator if

$$w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right).$$

Theorem 1: The *qHCPA* operator aggregates the "*n*" and again generates *qHCE*.

Proof: The proof is straightforward.

Remarks: Similarly, *qHCPWA*, *qHCPOWA*, *qHCPHA*, *qHCPG*, *qHCPWG*, *qHCPOWG* and *qHCPHG* also generates *qHCE*.

5. Some novel similarity measures for q-rung orthopair hesitant fuzzy connection numbers (qHCNs)

This section described the generalized form of similarity measures (SM) for *qHCNs*. Some characteristics are aslo explained with Jaccard SMs and Dice SMs. Similarity measures are vital tool for the similarity degree among sets.

Definition 20: [42] A real-valued function $s: x \times x \rightarrow [0,1]$ is called the similarity degree, if it follows the given properties for $R, S, T \in x$,

$$0 \le s(R,S) \le 1.$$
$$s(R,S) = 1 \Leftrightarrow R = S$$
$$s(R,S) = s(S,R)$$
$$\text{if } R \subseteq S \subseteq T \text{ then } s(R,S) \ge s(R,T)$$
$$\text{and } s(S,T) \ge s(R,T)$$

We straightforwardly developed Jaccard similarity measure and Dice similarity measure. In this part, we described SMs and weighted SMs (WSM) between the q-RHCNs based on the set theoretic approach for a collection of two q-RHCNs Z_1 and Z_2.

Definition 21: For two *qHCNs* Z_1 and Z_2, their Jaccard SMs are defined as:

$$Jac(Z_1, Z_2) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{E_{Z_1}(k)E_{Z_2}(k) + F_{Z_1}(k)F_{Z_2}(k) + G_{Z_1}(k)G_{Z_2}(k)}{\left(E_{Z_1}(k)\right)^2 + \left(E_{Z_2}(k)\right)^2 + \left(F_{Z_1}(k)\right)^2 + \left(F_{Z_2}(k)\right)^2 + \left(G_{Z_1}(k)\right)^2 + \left(G_{Z_2}(k)\right)^2 - \left(E_{Z_1}(k)E_{Z_2}(k) + F_{Z_1}(k)F_{Z_2}(k) + G_{Z_1}(k)G_{Z_2}(k)\right)}\right)$$

The JSMs for *qHCN* satisfy the following properties of SM:

$$0 \le Jac\,(Z_1, Z_2) \le 1$$
$$Jac\,(Z_1, Z_2) = Jac\,(Z_2, Z_1)$$
$$Jac\,(Z_1, Z_2) = 1 \Leftrightarrow Z_1 = Z_2$$

Definition 22: For two *qHCNs* Z_1 and Z_2, their weighted Jaccard JSM (WJSM) are defined as

$$Jac_w(Z_1, Z_2) = \sum_{i=1}^{n} w_i\left(\frac{E_{Z_1}(k)E_{Z_2}(k) + F_{Z_1}(k)F_{Z_2}(k) + G_{Z_1}(k)G_{Z_2}(k)}{\left(E_{Z_1}(k)\right)^2 + \left(E_{Z_2}(k)\right)^2 + \left(F_{Z_1}(k)\right)^2 + \left(F_{Z_2}(k)\right)^2 + \left(G_{Z_1}(k)\right)^2 + \left(G_{Z_2}(k)\right)^2 - \left(E_{Z_1}(k)E_{Z_2}(k) + F_{Z_1}(k)F_{Z_2}(k) + G_{Z_1}(k)G_{Z_2}(k)\right)}\right)$$

The *WJSM* for *qHCN* satisfies the following properties of SM:

$$0 \le Jac_w(Z_1, Z_2) \le 1$$
$$Jac_w(Z_1, Z_2) = Jac_w(Z_2, Z_1)$$
$$Jac_w(Z_1, Z_2) = 1 \Leftrightarrow Z_1 = Z_2$$

When we supposed the weight vector is

$$w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^T$$

at that point the WJSM would shift into JSM. Otherwise speaking when $w_k = \frac{1}{n}$, $k = 1, 2, 3, \dots n$ then $WJSM\,(Z_1, Z_2) = JSM\,(Z_1, Z_2)$

Definition 23: For two *qHCNs* Z_1 and Z_2, their Dice SMs (DSM) are defined as:

$$Dic(Z_1, Z_2) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{2E_{Z_1}(k).E_{Z_2}(k) + 2F_{Z_1}(k).F_{Z_2}(k) + 2G_{Z_1}(k).G_{Z_2}(k)}{\left(E_{Z_1}(k)\right)^2 + \left(E_{Z_2}(k)\right)^2 + \left(F_{Z_1}(k)\right)^2 + \left(F_{Z_2}(k)\right)^2 + \left(G_{Z_1}(k)\right)^2 + \left(G_{Z_2}(k)\right)^2}\right)$$

The DSM for *qHCN* satisfies the following properties of SM:

$$0 \le Dic\,(Z_1, Z_2) \le 1$$
$$Dic\,(Z_1, Z_2) = Dic\,(Z_2, Z_1)$$
$$Dic\,(Z_1, Z_2) = 1 \Leftrightarrow Z_1 = Z_2$$

Definition 24: For two *qHCNs* Z_1 and Z_2, their weighted DSM (WDSM) are defined as:

$$Dic_w(Z_1, Z_2) = \sum_{i=1}^{n} w_i\left(\frac{2E_{Z_1}(k).E_{Z_2}(k) + 2F_{Z_1}(k).F_{Z_2}(k) + 2G_{Z_1}(k).G_{Z_2}(k)}{\left(E_{Z_1}(k)\right)^2 + \left(E_{Z_2}(k)\right)^2 + \left(F_{Z_1}(k)\right)^2 + \left(F_{Z_2}(k)\right)^2 + \left(G_{Z_1}(k)\right)^2 + \left(G_{Z_2}(k)\right)^2}\right)$$

The WDSM for *qHCNs* satisfies the following properties of SM:

$$0 \le Dic_w\,(Z_1, Z_2) \le 1$$
$$Dic_w\,(Z_1, Z_2) = Dic_w\,(Z_2, Z_1)$$
$$Dic_w\,(Z_1, Z_2) = 1 \Leftrightarrow Z_1 = Z_2$$

When we assumed the weight vector is

$$w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^T$$

At that point the WDSM would change into DSM. Otherwise speaking when $w_k = \frac{1}{n}$, $k = 1, 2, 3, \dots n$ then $WDSM\,(Z_1, Z_2) = DSM\,(Z_1, Z_2)$.

6. An algorithm for decision-making techniques based on the structure of q-rung orthopair hesitant fuzzy connection numbers (qHCNs)

In this section, we employed a well-established approach that relies on similarity measures and *qHCNs* to make decisions. The information system has been developed with the guidance from experts. Similarity degree plays a crucial role in ranking both known and unknown alternatives.

Let us consider a unique set of alternatives denoted as *A*

= $A_1, A_2, \ldots, A_m$, which necessitates investigation with the support of a set of known alternatives represented by P. Additionally, we had a collection of attributes, denoted as $G = \{G_1, G_2, \ldots, G_n\}$. To facilitate our analysis, we introduced a weight vector for these attributes, denoted as $\omega = (\omega_t)$ for t ranging from 1 to n, where ω_t is non-negative, and the sum of all ω_t values equals 1. Therefore, we would apply the Jaccard weighted SM to find the similarity degrees.

The algorithm is briefly described step by step in Figure 1.

Step 1: Evaluate the known and unknown information of alternatives/attributes in the form of *qHCNs*.

Step 2: Develop the given data in the form of *qHCNs* according to the proposed approach in Definition 9.

Step 3: Calculate the similarity degrees between the known information and unknown information of alternatives.

Step 4: Rank all alternatives based on the aggregated similarity degrees and select the best one.

7. Applications in decision-making processes

The purpose of this section is to utilize the above developed similarity measures to resolve a real-life difficulty with q-RHF data.

Example 2:

Suppose that a counseling center has five various disciplines, namely History (G_1), Mathematics (G_2), Biology (G_3), Political studies (G_4), and Economics (G_5) related to career determination in the field of Politics (A_1), Pharmacy (A_2), Teaching (A_3) and Anatomy (A_4). Suppose that an unknown student P goes to the counseling center for getting assistance to select his/her appropriate profession. The purpose of the problem is to establish the most projected profession for the student P in the A_1, A_2, A_3 and A_4. To achieve this aim, the following stages of the suggested approach are developed, which are summarized as follows.

Step 1: A specialist provides the preference to each discipline connected to the profession, the q-RHFS were summarized in the following Table 1.

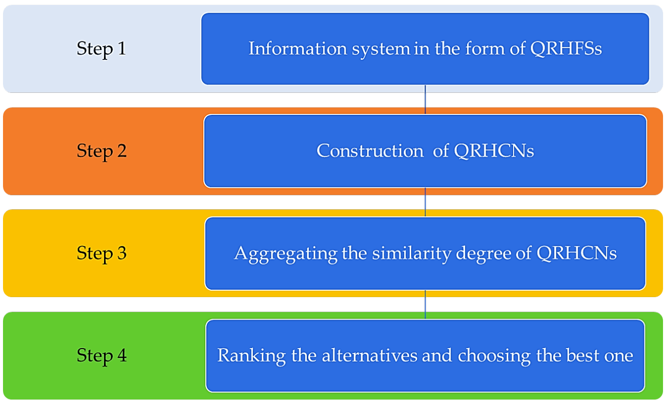

Figure 1. Flow chart of proposed algorithm

Table 1. q-RHFNs for all alternatives

Alter -natives	G_1	G_2	G_3	G_4	G_5
A_1	{0.1, 0.25},{0.16, 0.3}	{0.2, 0.1},{0.1, 0.4}	{0.3, 0.1},{0.5, 0.1}	{0.2, 0.14},{0.1, 0.12}	{0.25, 0.1},{0.5, 0.1}
A_2	{0.1, 0.12},{0.0, 0.2}	{0.12, 0.2},{0.3, 0.4}	{0.4, 0.1},{0.1, 0.13}	{0.1, 0.23},{0.1, 0.3}	{0.51, 0.4},{0.1, 0.2}
A_3	{0.3, 0.1},{0.2, 0.1}	{0.31 0.2},{0.1, 0.4}	{0.5, 0.1},{0.2, 0.3}	{0.01, 0.6},{0.12, 0.1}	{0.6, 0.2},{0.15, 0.2}
A_4	{0.4, 0.1},{0.3, 0.5}	{0.4, 0.5},{0.2, 0.1}	{0.1, 0.0},{0.1, 0.2}	{0.3, 0.1},{0.11, 0.2}	{0.31, 0.5},{0.12, 0.2}
P	{0.7, 0.3},{0.2, 0.1}	{0.7, 0.1},{0.1, 0.2}	{0.6, 0.4},{0.3, 0.1}	{0.4, 0.6},{0.15, 0.1}	{0.5, 0.4},{0.3, 0.2}

Step 2: *qHCNs* related to the preferences of the given data and student are prepared by applying Definition 9, and their results were summarized as Table 2.

Step 3: By allocating the weight vector $w = (0.15, 0.10. 0.20, 0.25, 0.30)^T$ corresponding to G_1, G_2, G_3, G_4 and G_5 then by using the recommended weighted similarity measure in Definition 14 and respectively, their corresponding measurement values are given $S(A_1P) = 0.035158$, $S(A_2P) = 0.387027$, $S(A_3P) = 0.190423$, $S(A_4P) = 0.183446$.

Step 4: From these calculated findings, we calculated the rank order of the alternatives as

$$A_2 > A_3 > A_4 > A_1,$$

Where ">" refers to "preferred to". A_2 is the best attribute for the students according to the proposed operator.

Example 3:

In this section, we illustrated the practical application of our established method in medical diagnosis through an example related to infectious diseases.

Considered a scenario where we had a set of patients, denoted as $A = \{A_1, A_2, A_3\}$, each of them required a medical diagnosis. We prepared a list of symptoms associated with potential diseases, represented in the form of q-RHF information. These symptoms are denoted as:

s = { Muscle pain (M), Temperature (T), Headache (H), Fever (F)}

Then, assumed we had a diagnosed patient, referred to as *P*, who presented with a specific set of symptoms. These symptoms served as valuable input data for determining the level of disease severity. By employing our method, we aimed to calculate the degree of similarity between the symptoms exhibited by patient *P* and those associated with various diseases, ultimately to assist in the diagnosis process.

Step 1: The q-RHF information of all patients are given in the Table 3.

Step 2: By definition 9 of *qHCN*, we received the alternatives values for $B = 2$ in Table 4.

Step 3: By allocating the weight vector $w = (0.5, 0.2. 0.2, 0.1)^T$ corresponding to *M*, *T*, *H* and *F* then by using the recommended weighted Jaccard similarity measure in definition 14 and respectively, their corresponding measurement values are given.

$S(A_1P) = 0.0107$, $S(A_2P) = 0.0084$, $S(A_3P) = 0.0073$

Step 4: From these calculated findings, we calculated the rank order of the alternatives as

$$A_1 > A_2 > A_3$$

From the above ranking, we could easily diagnose the status of patients by their symptoms. It is confirmed that A_1 was more similar to the *P* who was diagnosed patient.

Table 2. Obtained *qHCNs*

Alter -natives	G_1	G_2	G_3	G_4	G_5
A_1	0.129 + 0.686i +0.184j	0.120 + 0.655i +0.225j	0.120 + 0.660i +0.220j	0.151 + 0.756i +0.091j	0.107 + 0.660i +0.232j
A_2	0.098 + 0.814i +0.088j	0.102 + 0.618i +0.280j	0.2235 + 0.688i +0.0885j	0.125 + 0.714i +0.160j	0.389 + 0.526i +0.084j
A_3	0.165 + 0.720i +0.115j	0.199 + 0.6005i +0.2j	0.235 + 0.580i +0.185j	0.274 + 0.646i +0.079j	0.335 + 0.555i +0.11j
A_4	0.165 + 0.520i +0.315j	0.385 + 0.540i +0.075j	0.045 + 0.810i +0.145j	0.213 + 0.668i +0.118j	0.336 + 0.572i +0.091j
P	0.415 + 0.520i +0.065j	0.355 + 0.510i +0.135j	0.390+ 0.520i +0.090j	0.440 + 0.495i +0.065j	0.335 + 0.530i +0.135j

Table 3. q-RHF information of all patients

Alter -natives	M	T	H	F
A_1	{0.4, 0.3},{0.3, 0.6}	{0.2, 0.1},{0.7, 0.4}	{0.8, 0.1},{0.5, 0.4}	{0.2, 0.4},{0.1, 0.2}
A_2	{0.3, 0.5},{0.4, 0.2}	{0.8, 0.2},{0.3, 0.4}	{0.4, 0.1},{0.1, 0.7}	{0.1, 0.3},{0.1, 0.9}
A_3	{0.3, 0.4},{0.2, 0.4}	{0.7 0.2},{0.1, 0.4}	{0.5, 0.1},{0.8, 0.3}	{0.6, 0.6},{0.2, 0.1}
P	{0.7, 0.3},{0.2, 0.8}	{0.7, 0.1},{0.8, 0.2}	{0.6, 0.4},{0.8, 0.1}	{0.9, 0.6},{0.1, 0.1}

Table 4. *qHCNs*

Alter -natives	M	T	H	F
A_1	0.050 + 0.848i +0.100j	0.007 + 0.835i +0.157j	0.122 + 0.815i +0.062j	0.048 + 0.940i +0.010j
A_2	0.078 + 0.877i +0.043j	0.154 + 0.799i +0.046j	0.040 + 0.835i +0.123j	0.006 + 0.806i +0.186j
A_3	0.055 + 0.902i +0.042j	0.129 + 0.830i +0.039j	0.024 + 0.083i +0.142j	0.175 + 0.816i +0.008j
P	0.125 + 0.723i +0.150j	0.046 + 0.862i +0.091j	0.072+ 0.823i +0.104j	0.289 + 0.708i +0.002j

Table 5. Comparison analysis

Methods	Aggregation operators	Ranking
Garg et al.[41]	Distance Measures for IFCN	×
Tahir et al.[42]	IHPWA	$A_1 > A_2 > A_3$
	IHPWG	$A_1 > A_2 > A_3$
Wajid Ali et al.[28]	IHCPWA	$A_1 > A_2 > A_3$
	IHCPWG	$A_1 > A_2 > A_3$
Proposed approach	q-HCNPWA	$A_1 > A_2 > A_3$
	q-HCNPWG	$A_1 > A_2 > A_3$

7.1 Comparative analysis

In this section, we made a comparison between the existing studies and proposed model for the advancement and consistency of our structure. In addition, we conducted an in-depth comparison between the existing studies and our proposed model to highlight its advancements and consistency in structure. Subsequently, we discussed the advantages and limitations of our proposed model in detail.

We evaluated the performance of the proposed method by comparing it with current operators, such as power average aggregation and power geometric aggregation, specifically within the context of intuitionistic hesitant fuzzy data, as proposed by Tahir et al. [42], Garg et al. [41], and Wajid et al. [28]. For this comparative analysis, we utilized the data from example 2, and the corresponding results were presented in Table 5. This evaluation provided valuable insights into the effectiveness and efficiency of our approach relative to the existing methodologies.

In this analysis, we observed that the methods outlined by Tahir et al. [42] and Wajid et al. [28] were well-suited for an intuitionistic hesitant fuzzy environment. However, our proposed model stood out as it was more robust and versatile. Unlike the previous approaches, our model was capable of handling not only intuitionistic hesitant fuzzy environments but also pythagorean hesitant, fermatean hesitant and q-rung hesitant fuzzy environments.

Furthermore, by setting the parameter $B = 1$ our proposed method successfully reproduced all the results achieved by the approaches defined in [28, 42]. This consistency demonstrated the reliability and broader applicability of our developed model and made it a superior alternative for dealing with various types of hesitant fuzzy environments.

7.2 Advantages and limitation of the proposed approach

Some benefits and limitations of the proposed approach are listed as below:

- The proposed model serves as a generalized framework that extends both hesitant fuzzy sets and q-Rung orthopair fuzzy sets. By incorporating the connection number from set pair analysis, this model significantly enhances its ability to handle data uncertainty more effectively than existing approaches.
- The inclusion of the parameter B in the q-Rung hesitant connection number (q-HCN) model enables it to encompass various fuzzy environments, such as intuitionistic hesitant connection numbers, pythagorean hesitant connection numbers, fermatean hesitant connection numbers and square root fuzzy data.
- As demonstrated in Table 5, when the parameter $B = 1$, the results generated by the proposed model align closely with those of existing models, thereby confirming the correctness and consistency of the proposed approach.
- To facilitate data processing, the model included a series of aggregation operators, and several similarity

measures had been applied to address real-world problems, which showed the practical applicability of the model in diverse scenarios.

- However, the proposed model is limited to handling q-Rung fuzzy data. It did not extend to p,q-Rung orthopair fuzzy sets, where it failed to provide accurate results.
- Similarly, the model is not equipped to handle bipolar fuzzy information, thus made it unsuitable for p,q-Bipolar fuzzy data neither.

8. Conclusion

In this research, we introduced a novel concept: q-rung orthopair hesitant fuzzy sets, which was achieved by amalgamating q-rung orthopair fuzzy sets (q-ROFSs) with hesitant fuzzy sets (HFSs). q-rung orthopair hesitant fuzzy sets, denoted as q-RHFSs, represented a more versatile and potent tool for effectively managing situations characterized by ambiguity and uncertainty. We enriched this framework by integrating the notion of the connection number from set pair analysis into the q-rung orthopair hesitant fuzzy environment, thereby giving rise to an entirely new theoretical construct termed q-rung orthopair hesitant fuzzy connection number (*qHCN*). Furthermore, this research extended the scope of similarity measures, specifically Jaccard and Dice similarity measures, by introducing weighted versions of these measures. The salient properties of these novel measures had been thoroughly examined. We also presented a novel algorithm designed to calculate the attributes of alternatives by utilizing the weighted similarity measures to discern the characteristics of both known and unknown alternatives. An illustrative medical diagnosis problem had been successfully addressed by using this innovative approach, which highlighted its potential applications in the domains of pattern recognition, artificial intelligence and decision makings. We conducted an in-depth comparative study which incorporated the latest research findings and included a detailed analysis of both the benefits and limitations of the proposed model. This comprehensive evaluation provides a clearer understanding of how our model stands in relation to current advancements and identifies areas where it excels as well as where it may have constraints. Our future endeavors will encompass the application of this approach across various research domains, with a focus on addressing practical challenges. We intend to explore further extensions of fuzzy models [54-57], including bipolar fuzzy sets, complex fuzzy sets and cubic fuzzy models. Additionally, we will delve into neural network problems and endeavor to devise models rooted in optimization theory as part of our ongoing research agenda [58-59].

Conflict of interest

There is no conflict of interest.

Authors' contributions

Khawar Hassan contributed to the design and execution of experiments and research methodologies, Tanzeela Shaheen supervised this research and assisted in data collection and analysis. Wajid Ali focused on the theoretical framework and optimization processes. Iftikhar ul Haq provided critical insights into data interpretation and ensured the accuracy of results, while Nadia Bibi assisted with literature reviews, refining the analysis, and manuscript writing. Amal Kumar Adak contributed through computational analysis, technical support, and the development of algorithms.

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