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Original Research

Some aggregation operators of parameterized single-valued neutrosophic numbers and their MADM approach in simplified neutrosophic indeterminate scenarios

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Abstract: Simplified neutrosophic indeterminate sets/numbers (SNISs/SNINs) can conveniently express the partially determinate and partially indeterminate information of truth, falsity, indeterminacy membership degrees, which shows its main merit. Then, the existing aggregation algorithms of SNINs lack a parameterized aggregation method to reach a flexible decision-making method subject to different decision risks. Therefore, this study proposed a MADM approach using some aggregation operators of parameterized single-valued neutrosophic numbers in a simplified neutrosophic indeterminate scenario. First, we presented a conversion technique from SNINs to parameterized single-valued neutrosophic numbers (PSVNNs) by an indeterminate parameter. Second, we proposed the score function of PSVNN and the parameterized single-valued neutrosophic weighted arithmetic averaging (PSVNWAA) and parameterized single-valued neutrosophic weighted geometric averaging (PSVNWGA) operators. Third, a multiple attribute decision-making (MADM) approach with the smaller, moderate and larger decision risks is developed by the PSVNWAA or PSVNWGA operator in a SNIS setting. Forth, the developed MADM approach is applied to the choice problem of clinical physicians as a numerical example. Generally, the developed MADM approach reveals the advantage of flexible decision making subject to the smaller, moderate, and larger decision risks of decision makers in a neutrosophic indeterminate scenario. However, this study reveals the main contributions in the aggregation algorithms of PSVNNs and flexible MADM approach in a SNIS setting

Keywords: Neutrosophic indeterminate set, Parameterized single-valued neutrosophic number, Parameterized single-valued neutrosophic aggregation operator, Decision making

Introduction

As an extension of an (interval-valued) intuitionistic fuzzy set, a neutrosophic set (NS) [1] can be depicted independently by the membership functions of the truth, falsity, and indeterminacy in incomplete, inconsistent, and indeterminate environments. Since the membership functions lie in the nonstandard interval $]0^-, 1^+[$, NS is difficult to be applied to real applications. Thus, Ye [2] proposed a simplified NS (SNS), including single-valued NS (SVNS) and interval-valued NS (IVNS), as a subclass of NS. Since SNSs (SVNSs and IVNSs) easily express indeterminate and inconsistent information in real life problems, SNSs (SVNSs and IVNSs) have been widely used in various decision-making problems [3-19]. However, SNS cannot express the partial determinate and

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partial indeterminate information of the truth, falsity, and indeterminacy membership degrees in real applications.

It should be noted that a neutrosophic number (NN) [1, 20, 21] is composed of both its certain term and its indeterminate term. Then, the main advantage of NN is that it can flexibly represent the information of both the partial certainty and the partial uncertainty. Therefore, NNs have been used in multiple attribute decision making (MADM) problems in an indeterminate circumstance [22, 23]. However, NN cannot express the truth, falsity, and indeterminacy information.

In light of SNSs and NNs, Du et al. [24] first proposed the concept of simplified neutrosophic indeterminate sets/numbers (SNISs/SNINs) and developed a MADM approach using the SNIN aggregation operators subject to the indeterminate ranges of decision makers. Then, Lu et al. [25] introduced a MADM approach using the Einstein aggregation operators of SNINs with the indeterminate ranges of decision makers. However, the existing SNIN aggregation algorithms lack a parameterized aggregation algorithm to reach a flexible MADM method subject to various risks of decision makers, which shows the research gap. Motivated by the research gap, this study developed a MADM approach using some aggregation operators of parameterized single-valued neutrosophic numbers (PSVNNs) in the scenario of SNISs. To do so, we first presented a conversion technique from SNIN to the parameterized single-valued neutrosophic number (PSVNN) by an indeterminate parameter for specifying the indeterminate range. PSVNN can construct the family of single-valued neutrosophic numbers (SVNNs) regarding the specified values of the indeterminate parameter. Then, in view of the single-valued neutrosophic weighted arithmetic averaging (SVNWAA) and single-valued neutrosophic weighted geometric averaging (SVNWGA) operators [7], the parameterized SVNWAA (PSVNWAA) and parameterized SVNWGA (PSVNWGA) operators were introduced along with an indeterminate parameter. Then, a MADM approach with the smaller, moderate, and larger decision risks is developed by the PSVNWAA or PSVNWGA operator in a SNIS setting.

Generally, this study mainly reflected the following contributions:

(i)We presented a conversion technique from SNINs to PSVNNs by an indeterminate parameter.

(ii)We proposed the score function of PSVNN for ranking PSVNNs and the PSVNWAA and PSVNWGA operators for flexibly aggregating PSVNNs.

(iii)A MADM approach with the smaller, moderate, and larger decision risks is developed by the PSVNWAA or PSVNWGA operator in a SNIS setting.

(iv)The developed MADM approach is applied to the selection problem of clinical physicians in a hospital.

In this study, this paper contains the following structures. In Section 2, we briefly reviewed some basic concepts of SNISs and SNINs. Then, Section 3 introduces an indeterminate parameter $\lambda \in [0, 1]$ for specifying the indeterminate range in SNINs so as to transform SNINs into PSVNNs and defines the operational laws and score function of PSVNNs, and then proposes the PSVNWAA and PSVNWGA operators of PSVNNs in view of the SVNWAA and SVNWGA operators of SVNNs [7]. Section 4 develops a MADM approach with the smaller, moderate, and larger decision risks by the PSVNWAA or PSVNWGA operator in a SNIS setting. In Section 5, a decision-making example is presented to indicate the applicability and flexibility of the developed MADM approach. Finally, the conclusions and the next study are indicated in the section 6.

Basic concepts of SNISs and SNINs

To represent the hybrid information of both SNS and NNs, Du et al. [24] first presented the definition of SNIS when the membership functions of the truth, falsity, and indeterminacy in SNS are converted into the NN membership functions of the truth, falsity, and indeterminacy in a neutrosophic indeterminate setting.

Definition 1 [24]. Set X as a universe set. Then, a SNIS Z is defined as the following form:

$$Z = \{\langle x, \tilde{T}_Z(x,I), \tilde{U}_Z(x,I), \tilde{F}_Z(x,I)\rangle \mid x \in X, I \in [I^-, I^+]\} = \{\langle x, t+aI, u+bI, f+cI\rangle \mid x \in X, I \in [I^-, I^+]\}$$

which are independently depicted by the truth NN membership function $\tilde{T}_Z(x,I) = t + aI$ the indeterminacy NN membership function $\tilde{U}_Z(x,I) = u + bI$ and the falsity NN membership function $\tilde{F}_Z(x,I) = f + cI$ for $x \in X, I \in [I^-, I^+]$ $\tilde{T}_Z(x,I), \tilde{U}_Z(x,I), \tilde{F}_Z(x,I) \subseteq [0, 1]$ and t, u, f, a, b, c ∈ R (all real numbers) with the condition $0 \le \sup \tilde{T}_Z(x,I) + \sup \tilde{U}_Z(x,I) + \sup \tilde{F}_Z(x,I) \le 3$.

Then, the basic component $\langle x, \tilde{T}_Z(x,I), \tilde{U}_Z(x,I), \tilde{F}_Z(x,I)\rangle$ in the SNIS Z is denoted as $z(I) = \langle \tilde{T}(I), \tilde{U}(I), \tilde{F}(I)\rangle = \langle t+aI, u+bI, f+cI\rangle$ for the simplified representation and named a SNIN. It is obvious that SVNS or IVNS is a special case or a subclass of SNIS for a specified value or a specified interval number of $I \in [I^-, I^+]$.

Definition 2 [24]. Let $z_1(I) = \langle t_1 + a_1 I, u_1 + b_1 I, f_1 + c_1 I\rangle$ and $z_2(I) = \langle t_2 + a_2 I, u_2 + b_2 I, f_2 + c_2 I\rangle$ for $I \in [I^-, I^+]$. be two SNINs, then there are the following relations:

(i) $z_1(I) \subseteq z_2(I) \Leftrightarrow t_1 + a_1 I \subseteq t_2 + a_2 I$ $u_1 + b_1 I \supseteq u_2 + b_2 I$, and $f_1 + c_1 I \supseteq f_2 + c_2 I$.

(ii) $z_1(I) = z_2(I) \Leftrightarrow z_1(I) \subseteq z_2(I)$ and $z_2(I) \subseteq z_1(I)$ i.e., $t_1+a_1I=t_2+a_2I$, $u_1+b_1I=u_2+b_2I$, and $f_1+c_1I=f_2+c_2I$;

(iii) $z_1(I) \cup z_2(I) = \langle t_1 + a_1 I \cup t_2 + a_2 I, u_1 + b_1 I \cap u_2 + b_2 I, f_1 + c_1 I \cap f_2 + c_2 I\rangle$

(iv) $z_1(I) \cap z_2(I) = \langle t_1 + a_1 I \cap t_2 + a_2 I, u_1 + b_1 I \cup u_2 + b_2 I, f_1 + c_1 I \cup f_2 + c_2 I\rangle$

(v)
$z_1^c(I) = \langle [f_1 + c_1 I^-, f_1 + c_1 I^+], [1-(u_1+b_1 I^+), 1-(u_1+b_1 I^-)], [t_1 + a_1 I^-, t_1 + a_1 I^+] \rangle$
(the complement of z_1(I)).

PSVNN and aggregation operators of PSVNNs

For an indeterminate range of $I \in [I^-, I^+]$ in the SNIN $z(I) = \langle \tilde{T}(I), \tilde{U}(I), \tilde{F}(I) \rangle = \langle t + aI, u + bI, f + cI \rangle$ we can introduce an indeterminate parameter $\lambda \in [0, 1]$, so as to transform the SNIN z(I) into the following PSVNN:

$$z(\lambda) = \langle T(\lambda), U(\lambda), F(\lambda) \rangle = \langle t + aI^- + a\lambda(I^+ - I^-), u + bI^- + b\lambda(I^+ - I^-), f + cI^- + c\lambda(I^+ - I^-) \rangle \quad (1)$$

By similar operational laws of SVNNs [7], we can define the operational laws of PSVNNs below.

Definition 3. Let two PSVNNs be

$z_1(\lambda) = \langle T_1(\lambda), U_1(\lambda), F_1(\lambda) \rangle = \langle t_1 + a_1 I^- + a_1\lambda(I^+ - I^-), u_1 + b_1 I^- + b_1\lambda(I^+ - I^-), f_1 + c_1 I^- + c_1\lambda(I^+ - I^-) \rangle$

and
$z_2(\lambda) = \langle T_2(\lambda), U_2(\lambda), F_2(\lambda) \rangle = \langle t_2 + a_2 I^- + a_2\lambda(I^+ - I^-), u_2 + b_2 I^- + b_2\lambda(I^+ - I^-), f_2 + c_2 I^- + c_2\lambda(I^+ - I^-) \rangle$

for $\lambda \in [0, 1]$ and $I \in [I^-, I^+]$, we can define their basic operational laws:

(1)
$$z_1(\lambda) \oplus z_2(\lambda) = \langle T_1(\lambda) + T_2(\lambda) - T_1(\lambda)T_2(\lambda), U_1(\lambda)U_2(\lambda), F_1(\lambda)F_2(\lambda) \rangle = \left\langle \begin{array}{l} \left(t_1 + a_1 I^- + a_1\lambda(I^+ - I^-)\right) + \left(t_2 + a_2 I^- + a_2\lambda(I^+ - I^-)\right) \\ -\left(t_1 + a_1 I^- + a_1\lambda(I^+ - I^-)\right)\left(t_2 + a_2 I^- + a_2\lambda(I^+ - I^-)\right), \\ \left(u_1 + b_1 I^- + b_1\lambda(I^+ - I^-)\right)\left(u_2 + b_2 I^- + b_2\lambda(I^+ - I^-)\right), \\ \left(f_1 + c_1 I^- + c_1\lambda(I^+ - I^-)\right)\left(f_2 + c_2 I^- + c_2\lambda(I^+ - I^-)\right) \end{array} \right\rangle$$

(2)
$$z_1(\lambda) \otimes z_2(\lambda) = \langle T_1(\lambda)T_2(\lambda), U_1(\lambda) + U_2(\lambda) - U_1(\lambda)U_2(\lambda), F_1(\lambda) + F_2(\lambda) - F_1(\lambda)F_2(\lambda) \rangle = \left\langle \begin{array}{l} \left(t_1 + a_1 I^- + a_1\lambda(I^+ - I^-)\right)\left(t_2 + a_2 I^- + a_2\lambda(I^+ - I^-)\right), \\ \left(u_1 + b_1 I^- + b_1\lambda(I^+ - I^-)\right) + \left(u_2 + b_2 I^- + b_2\lambda(I^+ - I^-)\right) \\ -\left(u_1 + b_1 I^- + b_1\lambda(I^+ - I^-)\right)\left(u_2 + b_2 I^- + b_2\lambda(I^+ - I^-)\right), \\ \left(f_1 + c_1 I^- + c_1\lambda(I^+ - I^-)\right) + \left(f_2 + c_2 I^- + c_2\lambda(I^+ - I^-)\right) \\ -\left(f_1 + c_1 I^- + c_1\lambda(I^+ - I^-)\right)\left(f_2 + c_2 I^- + c_2\lambda(I^+ - I^-)\right) \end{array} \right\rangle$$

(3)
$$\delta z_1(\lambda) = \langle 1 - (1 - T_1(\lambda))^\delta, (U_1(\lambda))^\delta, (F_1(\lambda))^\delta \rangle = \left\langle \begin{array}{l} 1 - \left(1 - \left(t_1 + a_1 I^- + a_1\lambda(I^+ - I^-)\right)\right)^\delta, \\ \left(u_1 + b_1 I^- + b_1\lambda(I^+ - I^-)\right)^\delta, \\ \left(f_1 + c_1 I^- + c_1\lambda(I^+ - I^-)\right)^\delta \end{array} \right\rangle$$

for δ>0;

(4)
$$[z_1(\lambda)]^\delta = \langle (T_1(\lambda))^\delta, 1 - (1 - U_1(\lambda))^\delta, 1 - (1 - F_1(\lambda))^\delta \rangle = \left\langle \begin{array}{l} \left(t_1 + a_1 I^- + a_1\lambda(I^+ - I^-)\right)^\delta, \\ 1 - \left(1 - \left(u_1 + b_1 I^- + b_1\lambda(I^+ - I^-)\right)\right)^\delta, \\ 1 - \left(1 - \left(f_1 + c_1 I^- + c_1\lambda(I^+ - I^-)\right)\right)^\delta \end{array} \right\rangle$$

for δ > 0. For any PSVNN

$z(\lambda) = \langle T(\lambda), U(\lambda), F(\lambda) \rangle = \langle t + aI^- + a\lambda(I^+ - I^-), u + bI^- + b\lambda(I^+ - I^-), f + cI^- + c\lambda(I^+ - I^-) \rangle$

and $\lambda \in [0, 1]$, the score function of the PSVNN $z(\lambda)$ can be introduced based on the score function [7] as follows:

$$S(z(\lambda)) = \frac{1}{3}(2 + T(\lambda) - U(\lambda) - F(\lambda)) = \frac{1}{3}\begin{bmatrix} 2 + t + aI^- + a\lambda(I^+ - I^-) \\ -(u + bI^- + b\lambda(I^+ - I^-)) \\ -(f + cI^- + c\lambda(I^+ - I^-)) \end{bmatrix}, \quad (2)$$

$S(z(\lambda)) \in [0, 1]$, let

$z_j(\lambda) = \langle T_j(\lambda), U_j(\lambda), F_j(\lambda) \rangle = \langle t_j + a_j I^- + a_j\lambda(I^+ - I^-), u_j + b_j I^- + b_j\lambda(I^+ - I^-), f_j + c_j I^- + c_j\lambda(I^+ - I^-) \rangle$

(j = 1, 2, ..., n) for $\lambda \in [0, 1]$ be a group of PSVNNs. In view of the SVNWAA and SVNWGA operators of SVNNs [7], we can introduce the following PSVNWAA and PSVNWGA operators:

$$PSVNWAA(z_1(\lambda), z_2(\lambda), \ldots, z_n(\lambda)) = \sum_{j=1}^n w_j z_j(\lambda) = \langle 1 - \prod_{j=1}^n (1 - T_j(\lambda))^{w_j}, \prod_{j=1}^n (U_j(\lambda))^{w_j}, \prod_{j=1}^n (F_j(\lambda))^{w_j} \rangle = \left\langle \begin{array}{l} 1 - \prod_{j=1}^n (1 - (t_j + a_j I^- + a_j\lambda(I^+ - I^-)))^{w_j}, \\ \prod_{j=1}^n (u_j + b_j I^- + b_j\lambda(I^+ - I^-))^{w_j}, \\ \prod_{j=1}^n (f_j + c_j I^- + c_j\lambda(I^+ - I^-))^{w_j} \end{array} \right\rangle \quad (3)$$

$$PSVNWGA(z_1(\lambda), z_2(\lambda), \ldots, z_n(\lambda)) = \prod_{j=1}^n (z_j(\lambda))^{w_j} = \langle \prod_{j=1}^n (T_j(\lambda))^{w_j}, 1 - \prod_{j=1}^n (1 - U_j(\lambda))^{w_j}, 1 - \prod_{j=1}^n (1 - F_j(\lambda))^{w_j} \rangle = \left\langle \begin{array}{l} \prod_{j=1}^n (t_j + a_j I^- + a_j\lambda(I^+ - I^-))^{w_j}, \\ 1 - \prod_{j=1}^n (1 - (u_j + b_j I^- + b_j\lambda(I^+ - I^-)))^{w_j}, \\ 1 - \prod_{j=1}^n (1 - (f_j + c_j I^- + c_j\lambda(I^+ - I^-))^{w_j} \end{array} \right\rangle \quad (4)$$

where w_j (j = 1, 2, ..., n) is the weight of $z_j(\lambda)$ (j = 1, 2, ..., n) subject to $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Especially when $\lambda = 0$, the PSVNWAA and PSVNWGA operators are reduced to the SVNWAA and SVNWGA operators [7] as the special cases.

In light of the properties of the SVNWAA and SVNWGA operators [7], it is obvious that the PSVNWAA and PSVNWGA operators should also contain the following properties:

(1)Idempotency: Set

$$z_j(\lambda) = \langle T_j(\lambda), U_j(\lambda), F_j(\lambda)\rangle = \langle t_j + a_j I^- + a_j\lambda(I^+ - I^-), u_j + b_j I^- + b_j\lambda(I^+ - I^-), f_j + c_j I^- + c_j\lambda(I^+ - I^-)\rangle$$

($j = 1, 2, ..., n$) for $\lambda \in [0, 1]$ as a group of PSVNNs. If $z_j(\lambda) = z(\lambda)$, i.e. $z_j(\lambda) = z(\lambda) = <T(\lambda), U(\lambda), F(\lambda)>$, then $PSVNWAA(z_1(\lambda), z_2(\lambda), \ldots, z_n(\lambda) = z(\lambda)$ and $PSVNWGA(z_1(\lambda), z_2(\lambda), \ldots, z_n(\lambda) = z(\lambda)$.

(2)Boundedness: Set

$$z_j(\lambda) = \langle T_j(\lambda), U_j(\lambda), F_j(\lambda)\rangle = \langle t_j + a_j I^- + a_j\lambda(I^+ - I^-), u_j + b_j I^- + b_j\lambda(I^+ - I^-), f_j + c_j I^- + c_j\lambda(I^+ - I^-)\rangle$$

($j = 1, 2, ..., n$) for $\lambda \in [0, 1]$ as a group of PSVNNs, and then let the minimum and maximum PSVNNs:

$$z_{\min}(\lambda) = \left\langle \min_j\left(T_j(\lambda)\right), \max_j\left(U_j(\lambda)\right), \max_j\left(F_j(\lambda)\right)\right\rangle$$
$$= \left\langle \min_j\left(t_j + a_j I^- + a_j\lambda(I^+ - I^-)\right), \max_j\left(u_j + b_j I^- + b_j\lambda(I^+ - I^-)\right), \max_j\left(f_j + c_j I^- + c_j\lambda(I^+ - I^-)\right)\right\rangle$$

$$z_{\max}(\lambda) = \left\langle \max_j\left(T_j(\lambda)\right), \min_j\left(U_j(\lambda)\right), \min_j\left(F_j(\lambda)\right)\right\rangle$$
$$= \left\langle \max_j\left(t_j + a_j I^- + a_j\lambda(I^+ - I^-)\right), \min_j\left(u_j + b_j I^- + b_j\lambda(I^+ - I^-)\right), \min_j\left(f_j + c_j I^- + c_j\lambda(I^+ - I^-)\right)\right\rangle$$

Then, there exist

$z_{\min}(\lambda) \le PSVNWAA(z_1(\lambda), z_2(\lambda), \ldots, z_n(\lambda) \le z_{\max}(\lambda)$ and

$z_{\min}(\lambda) \le PSVNWGA(z_1(\lambda), z_2(\lambda), \ldots, z_n(\lambda) \le z_{\max}(\lambda)$.

(3)Monotonicity: set

$$z_j(\lambda) = \langle T_j(\lambda), U_j(\lambda), F_j(\lambda)\rangle = \langle t_j + a_j I^- + a_j\lambda(I^+ - I^-), u_j + b_j I^- + b_j\lambda(I^+ - I^-), f_j + c_j I^- + c_j\lambda(I^+ - I^-)\rangle$$

($j = 1, 2, \ldots, n$) for $\lambda \in [0, 1]$ as a group of PSVNNs, and then let $z_j(\lambda) \le z_j^*(\lambda)$ Thus, there exist the inequalities $PSVNWAA(z_1(\lambda), z_2(\lambda), \ldots, z_n(\lambda) \le PSVNWAA(z_1^*(\lambda), z_2^*(\lambda), \ldots, z_n^*(\lambda)$ and $PSVNWGA(z_1(\lambda), z_2(\lambda), \ldots, z_n(\lambda) \le PSVNWGA(z_1^*(\lambda), z_2^*(\lambda), \ldots, z_n^*(\lambda)$.

A MADM approach using the PSVNWAA or PSVNWGA operator

This section develops a MADM approach using the PSVNWAA or PSVNWGA operator in a SNIS setting.

Suppose that a MADM problem contains a set of m alternatives $M = \{M_1, M_2, \ldots, M_s\}$ and a set of attributes $R = \{R_1, R_2, \ldots, R_n\}$. Then the weigh vector of $R = \{R_1, R_2, \ldots, R_n\}$ are specified as $w = (w_1, w_2, \ldots, w_n)$. Thus, the alternatives M_i ($i = 1, 2, \ldots, s$) are evaluated over the attributes R_j($j = 1, 2, \ldots, n$) by the SNIN

$$z_{ij}(I) = \left\langle \tilde{T}_{ij}(I), \tilde{U}_{ij}(I), \tilde{F}_{ij}(I)\right\rangle = \left\langle t_{ij} + a_{ij}I, u_{ij} + b_{ij}I, f_{ij} + c_{ij}I\right\rangle$$

for $I \in [I^-, I^+]$, $\lambda \in [0, 1]$, $\tilde{T}_{ij}, \tilde{U}_{ij}, \tilde{F}_{ij} \subseteq [0, 1]$ and t_{ij}, u_{ij}, f_{ij}, a_{ij}, b_{ij}, $c_{ij} \in R$ ($i = 1, 2, \ldots, s$; $j = 1,2, \ldots, n$). Therefore, all SNINs given by decision makers can be constructed as the decision matrix of SNINs $Z(I) = (z_{ij}(I))_{s\times n}$.

Regarding a MADM problem in the setting of SNINs, we give the following MADM approach using the PSVNWAA or PSVNWGA operator subject to the decision risks of decision makers, which is presented as the following decision steps.

Step 1: Using Eq. (1) with an indeterminate parameter $\lambda \in [0, 1]$, each SNIN

$$z_{ij}(I) = \left\langle \tilde{T}_{ij}(I), \tilde{U}_{ij}(I), \tilde{F}_{ij}(I)\right\rangle = \left\langle t_{ij} + a_{ij}I, u_{ij} + b_{ij}I, f_{ij} + c_{ij}I\right\rangle$$

($i = 1, 2, \ldots, s$; $j = 1,2, \ldots, n$) for $I \in [I^-, I^+]$ is transformed into the following PSVNN:

$$z_{ij}(\lambda) = \langle T_{ij}(\lambda), U_{ij}(\lambda), F_{ij}(\lambda)\rangle = \langle t_{ij} + a_{ij} I^- + a_{ij}\lambda(I^+ - I^-), u_{ij} + b_{ij} I^- + b_{ij}\lambda(I^+ - I^-), f_{ij} + c_{ij} I^- + c_{ij}\lambda(I^+ - I^-)\rangle \quad (5)$$

Then, all PSVNNs can be constructed as their decision matrix $Z(\lambda) = (z_{ij}(\lambda))_{s\times n}$.

Step 2: On the basis of Eq. (3) or (4) along with the smaller decision risk ($\lambda = 0$) or the moderate decision risk (λ= 0.5) or the larger decision risk ($\lambda = 1$) in a neutrosophic indeterminate environment, the aggregation values of the PSVNNs $z_{ij}(\lambda)$ for M_i ($i = 1, 2, \ldots, s$) are given by the following formula:

$$z_i(\lambda) = PSVNWAA(z_{i1}(\lambda), z_{i2}(\lambda), \ldots, z_{in}(\lambda)) = \sum_{j=1}^{n} w_j z_{ij}(\lambda) =$$
$$\langle 1 - \prod_{j=1}^{n}(1 - T_{ij}(\lambda))^{w_j}, \prod_{j=1}^{n}(U_{ij}(\lambda))^{w_j}, \prod_{j=1}^{n}(F_{ij}(\lambda))^{w_j}\rangle =$$
$$\left\langle \begin{array}{c} 1 - \prod_{j=1}^{n}\left(1 - (t_{ij} + a_{ij}I^- + a_{ij}\lambda(I^+ - I^-))\right)^{w_j}, \\ \prod_{j=1}^{n}(u_{ij} + b_{ij}I^- + b_{ij}\lambda(I^+ - I^-))^{w_j}, \\ \prod_{j=1}^{n}(f_{ij} + c_{ij}I^- + c_{ij}\lambda(I^+ - I^-))^{w_j} \end{array}\right\rangle \quad (6)$$

or

$$z_i(\lambda) = PSVNWGA(z_{i1}(\lambda), z_{i2}(\lambda), \ldots, z_{in}(\lambda)) = \prod_{j=1}^{n}(z_{ij}(\lambda))^{w_j} =$$
$$\langle \prod_{j=1}^{n}(T_{ij}(\lambda))^{w_j}, 1 - \prod_{j=1}^{n}(1 - U_{ij}(\lambda))^{w_j}, 1 - \prod_{j=1}^{n}(1 - F_{ij}(\lambda))^{w_j}\rangle$$
$$= \left\langle \begin{array}{c} \prod_{j=1}^{n}(t_{ij} + a_{ij}I^- + a_{ij}\lambda(I^+ - I^-))^{w_j}, \\ 1 - \prod_{j=1}^{n}\left(1 - (u_{ij} + b_{ij}I^- + b_{ij}\lambda(I^+ - I^-))\right)^{w_j}, \\ 1 - \prod_{j=1}^{n}\left(1 - (f_{ij} + c_{ij}I^- + c_{ij}\lambda(I^+ - I^-)\right)^{w_j} \end{array}\right\rangle \quad (7)$$

Step 3:, The score values of $S(z_1(\lambda))$ (i = 1, 2, …, s) are computed by Eq. (2).

Step 4: The alternatives are ranked in a descending order based on the score values, and the best is selected regarding the alternative with the highest score value.

Numerical example and comparative analysis

Numerical example

This subsection provides a MADM example in a SNIS setting to illustrate the applicability of the developed MADM approach.

Some hospital would like to recruit a clinical physician. Through the human resources department, the four candidates (the four alternatives) M_1, M_2, M_3, and M_4 are preliminarily selected from all the applicants, the set of four candidates (alternatives) $M = \{M_1, M_2, M_3, M_4\}$ must be assessed with respect to three requirements (attributes): (a) R_1 is clinical skill; (b) R_2 is previous clinical experience; (c) R_3 is self-confidence. Experts are invited to make the interview and to select the best candidate by the suitable assessment of them. Then, the weighted vector of the three requirements is $\boldsymbol{w}$ = (0.4, 0.35, 0.25), which is specified by the decision makers. In this assessment problem, the assessment values of the three requirements for the four candidates are given in the form of SNINs for the indeterminacy $I \in [0, 1]$ and constructed as the following decision matrix of SNINs $Z(I) = (z_{ij}(I))_{4\times3}$:

$$Z(I)=\begin{array}{c}M_1\\M_2\\M_3\\M_4\end{array}\begin{bmatrix}\langle 0.6+0.15I,0.2+0.1I,0.2+0.2I\rangle & \langle 0.6+0.2I,0.3+0.2I,0.2+0.1I\rangle & \langle 0.6+0.15I,0.2+0.2I,0.2+0.1I\rangle\\ \langle 0.7+0.2I,0.3+0.2I,0.4+0.1I\rangle & \langle 0.6+0.2I,0.2+0.2I,0.5+0.15I\rangle & \langle 0.6+0.1I,0.3+0.1I+,0.3+0.1I\rangle\\ \langle 0.7+0.1I,0.4+0.1I,0.3+0.2I\rangle & \langle 0.8+0.2I,0.3+0.2I,0.4+0.1I\rangle & \langle 0.7+0.2I,0.3+0.1I,0.4+0.1I\rangle\\ \langle 0.7+0.2I,0.4+0.1I,0.5+0.1I\rangle & \langle 0.7+0.2I,0.3+0.1I,0.2+0.1I\rangle & \langle 0.6+0.15I,0.4+0.1I,0.5+0.1I\rangle\end{bmatrix}$$

Thus, the developed MADM approach with the smaller decision risk (λ = 0) or the moderate decision risk (λ = 0.5) or the larger decision risk (λ = 1) in the scenario of SNISs is used for this MADM problem with the SNIN information, which is described in the decision steps below.

Step 1: By Eq. (5) with an indeterminate parameter $\lambda \in [0, 1]$, the decision matrix of SNINs $Z(I) = (z_{ij}(I))_{4\times3}$ for $I \in [0, 1]$ is transformed into the following decision matrix of PSVNNs:

$$Z(\lambda)=\begin{array}{c}M_1\\M_2\\M_3\\M_4\end{array}\begin{bmatrix}\langle 0.6+0.15\lambda,0.2+0.1\lambda,0.2+0.2\lambda\rangle & \langle 0.6+0.2\lambda,0.3+0.2\lambda,0.2+0.1\lambda\rangle & \langle 0.6+0.15\lambda,0.2+0.2\lambda,0.2+0.1\lambda\rangle\\ \langle 0.7+0.2\lambda,0.3+0.2\lambda,0.4+0.1\lambda\rangle & \langle 0.6+0.2\lambda,0.2+0.2\lambda,0.5+0.15\lambda\rangle & \langle 0.6+0.1\lambda,0.3+0.1\lambda+,0.3+0.1\lambda\rangle\\ \langle 0.7+0.1\lambda,0.4+0.1\lambda,0.3+0.2\lambda\rangle & \langle 0.8+0.2\lambda,0.3+0.2\lambda,0.4+0.1\lambda\rangle & \langle 0.7+0.2\lambda,0.3+0.1\lambda,0.4+0.1\lambda\rangle\\ \langle 0.7+0.2\lambda,0.4+0.1\lambda,0.5+0.1\lambda\rangle & \langle 0.7+0.2\lambda,0.3+0.1\lambda,0.2+0.1\lambda\rangle & \langle 0.6+0.15\lambda,0.4+0.1\lambda,0.5+0.1\lambda\rangle\end{bmatrix}$$

Step 2: On the basis of Eq. (6) or Eq. (7) along with the smaller decision risk (λ = 0) or the moderate decision risk (λ = 0.5) or the larger decision risk (λ = 1),the PSVNN aggregation values $z_i(\lambda)$ for M_i (i = 1, 2, …, s) are shown in Table 1.

Step 3: By Eq. (2), the score values of $S(z_i(\lambda))$ (i = 1, 2, …, s) regarding the smaller decision risk (λ = 0) or the moderate decision risk (λ = 0.5) or the larger decision risk (λ = 1) are also given in Table 1.

Step 4: The ranking orders of the four candidates (alternatives) are also tabulated in Table 1, where the symbol "$\succ$" means "superior to".

Table 1. Decision results of the proposed MADM approach with decision risks

Aggregation operator	Decision risk λ	$z_1(\lambda)$	$z_2(\lambda)$	$z_3(\lambda)$	$z_4(\lambda)$	$S(z_i(\lambda))$	Ranking order
PSVNWAA	Smaller decision risk (λ = 0)	<0.6000, 0.2305, 0.2000>	<0.6435, 0.2603, 0.4025>	<0.7397, 0.3366, 0.3565>	<0.6776, 0.3617, 0.3628>	0.7232, 0.6602, 0.6822, 0.6510	$M_1 \succ M_3 \succ M_2 \succ M_4$
	Moderate decision risk (λ = 0.5)	<0.6840, 0.3084, 0.2689>	<0.7349, 0.3498, 0.4605>	<0.8284, 0.4055, 0.4293>	<0.7742, 0.4121, 0.4174>	0.7022, 0.6415, 0.6645, 0.6482	$M_1 \succ M_3 \succ M_4 \succ M_2$
	Larger decision risk (λ = 1)	<0.7688, 0.3855, 0.3366>	<0.8323, 0.4373, 0.5183>	<1.0000, 0.4729, 0.5000>	<0.8743, 0.4624, 0.4708>	0.6822, 0.6255, 0.6757, 0.6470	$M_1 \succ M_3 \succ M_4 \succ M_2$
PSVNWGA	Smaller decision risk (λ = 0)	<0.6000, 0.2365, 0.2000>	<0.6382, 0.2665, 0.4150>	<0.7335, 0.3419, 0.3618>	<0.6735, 0.3667, 0.4106>	0.7212, 0.6522, 0.6766, 0.6321	$M_1 \succ M_3 \succ M_2 \succ M_4$
	Moderate decision risk (λ = 0.5)	<0.6836, 0.3182, 0.2704>	<0.7249, 0.3539, 0.4760>	<0.8124, 0.4088, 0.4305>	<0.7667, 0.4169, 0.4619>	0.6983, 0.6316, 0.6577, 0.6293	$M_1 \succ M_3 \succ M_2 \succ M_4$
	Larger decision risk (λ = 1)	<0.7671, 0.4013, 0.3419>	<0.8111, 0.4422, 0.5381>	<0.8908, 0.4767, 0.5000>	<0.8599, 0.4671, 0.5135>	0.6747, 0.6103, 0.6381, 0.6265	$M_1 \succ M_3 \succ M_4 \succ M_2$

From the decision results of Table 1, different decision risks can result in different ranking orders of the four alternatives. Then, the best candidate is M_1.

Sensitivity analysis

To show the sensitivity of the ranking orders to the values of the indeterminate parameter λ, we give the score values of the four alternatives based on the developed MADM approach using the PSVNWAA and PSVNWGA operators corresponding to $\lambda \in [0, 1]$, which are shown in Figures 1 and 2.

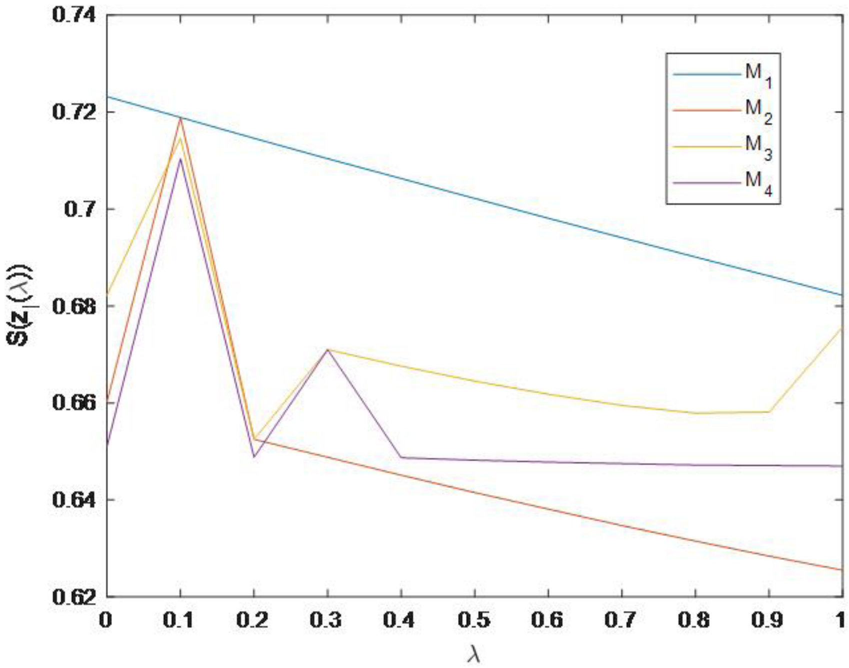

Figure 1. Score values of the four alternatives based on the developed MADM approach using the PSVNWAA operator corresponding to $\lambda \in [0, 1]$

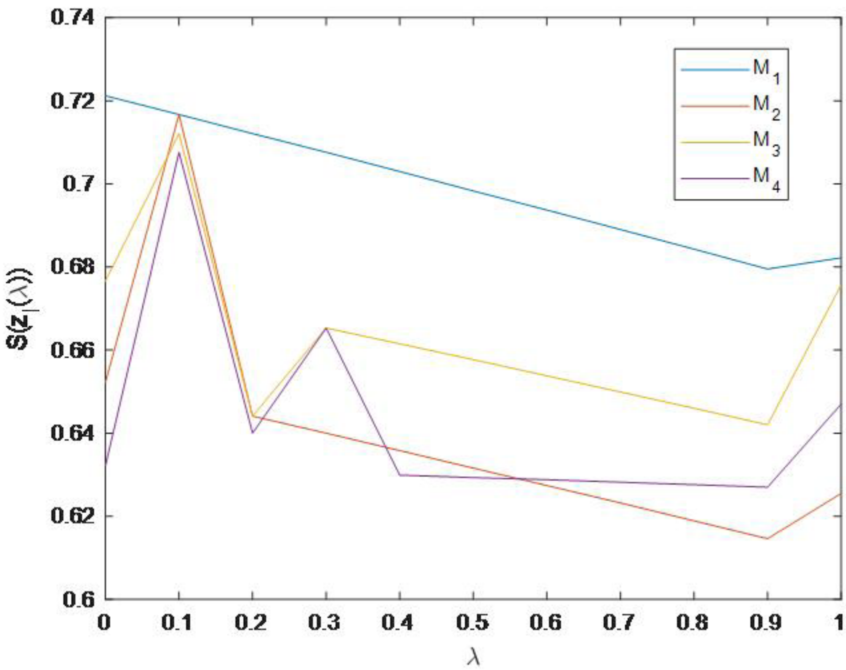

Figure 2. Score values of the four alternatives based on the developed MADM approach using the PSVNWGA operator corresponding to $\lambda \in [0, 1]$

In view of the score values in Figures 1 and 2, it is obvious that the ranking orders of the four alternatives are changed with the change of , which demonstrates sensitivity to λ.

Comparative analysis

To indicate the relative comparison, the existing MADM approach based on the SVNWAA and SVNWGA operators [7] can be applied to this numerical example when we considerate a special case of the SNIN decision matrix $Z(I)$ for $I = 0$. Thus, the decision results of the existing MADM approach [7] are tabulated in Table 2.

Table 2. Decision results of the existing MADM approach [7]

Aggre-gation operator	$z_1(\lambda)$	$z_2(\lambda)$	$z_3(\lambda)$	$z_4(\lambda)$	$S(z_i(\lambda))$	Ranking order
SVNWAA [7]	<0.6000, 0.2305, 0.2000>	<0.6435, 0.2603, 0.4025>	<0.7397, 0.3366, 0.3565>	<0.6776, 0.3617, 0.3628>	0.7232, 0.6602, 0.6822, 0.6510	$M_1 \succ M_3 \succ M_2 \succ M_4$
SVNWGA [7]	<0.6000, 0.2365, 0.2000>	<0.6382, 0.2665, 0.4150>	<0.7335, 0.3419, 0.3618>	<0.6735, 0.3667, 0.4106>	0.7212, 0.6522, 0.6766, 0.6321	$M_1 \succ M_3 \succ M_2 \succ M_4$

Based on the decision results in Tables 1 and 2, the existing MADM approach using the SVNWAA or SVNWGA operator is a special case of the developed MADM approach using the PSVNWAA or PSVNWGA operator for $\lambda = 0$ (the smaller decision risk). However, the developed MADM approach with different decision risks ($\lambda = 0, 0.5, 1$) can result in different ranking orders: $M_1 \succ M_3 \succ M_2 \succ M_4$ and $M_1 \succ M_3 \succ M_4 \succ M_2$, which indicate the flexibility of the developed MADM approach depending on the decision risks/indeterminate degrees of decision makers; while the existing MADM approach [7] lacks its decision flexibility. Then, the best candidate is M1 in all the decision results.

On the other hand, since the PSVNN contains a family of SVNNs for $\lambda \in [0, 1]$ and $I \in [I^-, I^+]$, the developed MADM approach demonstrates its flexible decision results depending on the decision risks ($\lambda = 0, 0.5, 1$) of decision makers and the indeterminate range of $I \in [I^-, I^+]$; while the existing MADM approaches [7, 24, 25] neither express the PSVNN information nor contain the decision risks, which shows their defects. However, the developed MADM approach can overcome the defects of the existing MADM approaches. It is obvious that the developed MADM approach is superior to the existing MADM approaches in a neutrosophic indeterminate setting.

Generally, the developed MADM approach demonstrates the obvious advantages of the parameterized aggregation algorithms and flexible decision-making method subject to various risks of decision makers in a neutrosophic indeterminate setting.

Conclusion

Regarding SNIS/SNIN information in indeterminate and inconsistent problems, this paper first presented a conversion technique from SNINs to PSVNNs by an indeterminate parameter $\lambda \in [0, 1]$ for specifying the indeterminate range of $I \in [I^-, I^+]$. Thus, PSVNN with respect to various specified values of $\lambda \in [0, 1]$ can construct the family of SVNNs. Meanwhile, the score function of PSVNN and the PSVNWAA and PSVNWGA operators of PSVNNs were introduced in view of the existing SVNWAA and SVNWGA operators of SVNNs. Next, a MADM approach with the smaller, moderate and larger decision risks ($\lambda = 0, 0.5, 1$) was developed by the PSVNWAA or PSVNWGA operator in a SNIS setting. Lastly, a numerical example was presented to indicate the applicability and flexibility of the developed MADM approach. However, the advantage of the developed MADM approach is that it can flexibly solve MADM problems with SNIN information regarding different decision risks in a neutrosophic indeterminate setting and avoid the defects of existing MADM approaches.

Since this study only presented the PSVNWAA and PSVNWGA operators (the most basic aggregation algorithms) and their MADM approach, they cannot capture the interrelationship between several input arguments in the decision-making process, which shows their limitation. In the future study, we shall further develop some good aggregation operators (e.g., Bonferroni aggregation and Hamacher aggregation operators of SNINs) and their applications in group decision making, medical diagnosis, pattern recognition, and clustering analysis in a SNIS setting.

Conflict of interest

The authors declare no conflict of interest.

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